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WAKE FIELD ENERGY SPREAD AND MICROWAVE INSTABILITY

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ABSTRACT

Microwave instability is still an open subject not totally understood. The common theory regards the phenomenon as a coupling of coherent modes of oscillation that generates instability. The wake fields are responsible of this coupling and cause, as a consequence, an increase of energy spread. According to the theory, there is a threshold below which the wake fields do not change the energy spread but only the bunch length. In this paper we assume that wake fields can produce a heating of the bunch, thus increasing the energy spread even below threshold. This effect does not preclude the existence of mode coupling instability.

1 Introduction

Microwave instability [1)] is still an open subject not completely understood. The theory commonly used [2,3)] considers the phenomenon as produced by the wake fields responsible of a coupling of longitudinal (azimuthal or radial [4)]) coherent modes of oscillation. When the modes couple together, they become unstable, causing an increase of the energy spread. The theory predicts a threshold below which the modes are stable, and the energy spread is not affected by the wake fields, that change only the bunch length by means of potential well distortion [5)]. However, so far, there is no experimental evidence of longitudinal mode coupling, differently from what observed in the transverse case. Moreover numerical estimates of the threshold by using mode coupling theory do not always agree with measurements [6)].

Recently, a bunch lengthening model relying on quantum fluctuation effects related to wake fields has been presented [7)]. Proceeding analogously to that model, in this paper we assume that wake fields could produce a heating of the bunch, thus increasing the energy spread even below threshold. The physical reason of this heating could be traced back, for example, to the strong nonlinearities of the wake fields. It is important to note that this hypothesis does not preclude the existence of mode coupling instability.

In the following section we calculate the variance of the bunch energy obtaining the expression of energy spread induced by the wake fields. This energy spread is summed quadratically to the natural energy spread, and the new value is used in section 3 to obtain bunch length and energy spread vs current for simple cases: pure resistive, pure inductive, and combined resistive - inductive wake fields. In section 4 the analytical results are compared with measurements.

2 Energy spread induced by wake fields

The energy lost in one turn by a particle in the position z_i due to wake fields produced by the whole bunch can be written as

$$U_i = e^2 N_p \int_{-\infty}^{\infty} dz' \rho(z') w(z' - z_i) \tag{1}$$

where N_p is the number of particle per bunch, $\rho(z)$ is the normalised longitudinal distribution function, and $w(z)$ the longitudinal wake fields.

From equation (1) it is possible to calculate the average energy lost in one turn weighted over the bunch distribution

$$\langle U \rangle = e^2 N_p \int_{-\infty}^{\infty} dz_i \rho(z_i) \int_{-\infty}^{\infty} dz' \rho(z') w(z' - z_i) \tag{2}$$

and the consequent energy variance, that, by definition, is given by

$$\begin{aligned}\left\langle E^2 \right\rangle = \left\langle [\langle U \rangle - U_i]^2 \right\rangle = e^4 N_p^2 \left[\int_{-\infty}^{\infty} dz_i \rho(z_i) \int_{-\infty}^{\infty} dz' \rho(z') w(z' - z_i) \right]^2 \\ -e^4 N_p^2 \int_{-\infty}^{\infty} dz_i \rho(z_i) \left[\int_{-\infty}^{\infty} dz' \rho(z') w(z' - z_i) \right]^2\end{aligned} \tag{3}$$

Equation (3) gives the energy variance for general expressions of wake fields and longitudinal distribution. If we treat this energy variance as energy noise, the related energy spread is [8)]

$$\sigma_{\varepsilon w}^2 = \frac{\left\langle E^2 \right\rangle}{2DE_o^2} \tag{4}$$

with D the damping coefficient, and E_o the nominal energy. By introducing the natural energy spread $\sigma_{\varepsilon o}$, we can write the total energy spread as

$$\sigma_\varepsilon^2 = \sigma_{\varepsilon o}^2 + \sigma_{\varepsilon w}^2 \tag{5}$$

3 Resistive and inductive cases

The value of $\sigma_{\varepsilon w}$ for a Gaussian distribution in case of pure resistive wake fields of the kind

$$w(z) = cR_s\delta(z) \tag{6}$$

with c speed of light, R_s the shunt resistance and $\delta(z)$ the delta function, can be easily calculated from equations (3) and (4). Since the potential well distortion for resistive wake fields does not affect very much the bunch length, we assume that the changes in σ_z are only due to the energy spread, that is

$$\sigma_z = \sqrt{\sigma_{zo}^2 + \sigma_{zw}^2} \qquad \sigma_{zw} = \frac{\alpha_c c}{\omega_s}\sigma_{\varepsilon w} \tag{7}$$

with σ_{zo} the natural bunch length, α_c the momentum compaction, and ω_s the synchrotron frequency. By substituting in equation (7) the expression of $\sigma_{\varepsilon w}$, we obtain

$$\sigma_z^4 - \sigma_{zo}^2\sigma_z^2 - \frac{\pi\alpha_c^2 I_b^2 R_s^2}{v_s^2 (E_o/e)^2 D}\left(\frac{1}{\sqrt{3}} - \frac{1}{2}\right) R_o^4 = 0 \tag{8}$$

$$\sigma_\varepsilon^2 = \sigma_{\varepsilon o}^2 + \frac{\pi I_b^2 R_s^2}{(E_o/e)^2 D}\left(\frac{1}{\sqrt{3}} - \frac{1}{2}\right)\left(\frac{R_o}{\sigma_z}\right)^2 \tag{9}$$

where I_b is the bunch current, ν_s the synchrotron tune, and R_o the machine radius. Equation (9) has to be solved once σ_z is known from (8).

Pure inductive wake fields can be written as

$$w(z) = -cR_o (Z/n)_i \delta'(z) \tag{10}$$

with $\delta'(z)$ the derivative of $\delta(z)$. As for resistive wake fields, we can calculate the energy spread $\sigma_{\varepsilon w}$ and, with potential well distortion theory and parabolic bunch distribution [9], we obtain

$$\sigma_z{}^6 - \sigma_{zo}^2 \sigma_z{}^4 - \frac{3\alpha_c I_b (Z/n)_i}{4\sqrt{2}\nu_s^2 (E_o/e)} R_o^3 \sigma_z{}^3 - \frac{\pi \alpha_c^2 I_b^2 (Z/n)_i^2}{3\sqrt{3}\nu_s^2 (E_o/e)^2 D} R_o^6 = 0 \tag{11}$$

$$\sigma_\varepsilon^2 = \sigma_{\varepsilon o}^2 + \frac{\pi I_b^2 (Z/n)_i^2}{3\sqrt{3}(E_o/e)^2 D} \left(\frac{R_o}{\sigma_z}\right)^4 \tag{12}$$

When we combine the resistive and inductive cases, and assume that only the inductive component produces potential well distortion, but both the wake fields change the energy spread, we end up with the following expressions

$$\begin{aligned} \sigma_z^6 - \sigma_{zo}^2 \sigma_z^4 - \frac{3\alpha_c I_b (Z/n)_i}{4\sqrt{2}\nu_s^2 (E_o/e)} R_o^3 \sigma_z^3 - \frac{\pi \alpha_c^2 I_b^2 R_s^2}{\nu_s^2 (E_o/e)^2 D}\left(\frac{1}{\sqrt{3}} - \frac{1}{2}\right) R_o^4 \sigma_z^2 \\ - \frac{\pi \alpha_c^2 I_b^2 (Z/n)_i^2}{3\sqrt{3}\nu_s^2 (E_o/e)^2 D} R_o^6 = 0 \end{aligned} \tag{13}$$

$$\sigma_\varepsilon^2 = \sigma_{\varepsilon o}^2 + \frac{\pi I_b^2 R_s^2}{(E_o/e)^2 D}\left(\frac{1}{\sqrt{3}} - \frac{1}{2}\right)\left(\frac{R_o}{\sigma_z}\right)^2 + \frac{\pi I_b^2 (Z/n)_i^2}{3\sqrt{3}(E_o/e)^2 D}\left(\frac{R_o}{\sigma_z}\right)^4 \tag{14}$$

that give the behaviour of bunch length and energy spread as a function of current.

4 Comparison with measurements

In order to get the values of σ_z and σ_ε, it is necessary to know R_s and $(Z/n)_i$. The real part of the machine impedance can be calculated by measuring the synchrotron phase shift as a function of current, as we have done for DAΦNE Accumulator Ring [10], for which the corresponding loss factor is 2.57*10^{11} V/C, and therefore $R_s = 150\ \Omega$ (with

$\sigma_z = 5$ cm). Then by fitting the bunch length with the scaling law [11], the absolute value of Z/n can be obtained. For DAΦNE Accumulator we got $|Z/n| = 3.5\,\Omega$, and, as a consequence, $(Z/n)_i = 3.2\ \Omega$. The values of R_s and $(Z/n)_i$ have been used in equations (13) and (14) to obtain the curves of figure 1, where we have reported also the measurements and the results of a simulation code [10] as comparison. The agreement is fairly good in the case of bunch length, while, for the energy spread, there are no experimental data.

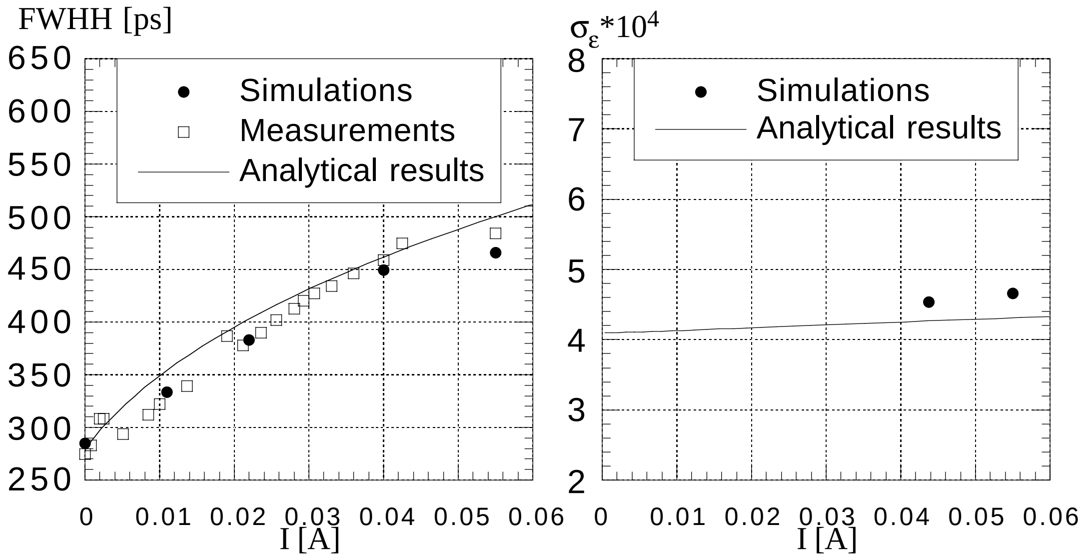

Figure 1: *Full width half high and energy spread vs current for DAΦNE Accumulator Ring.*

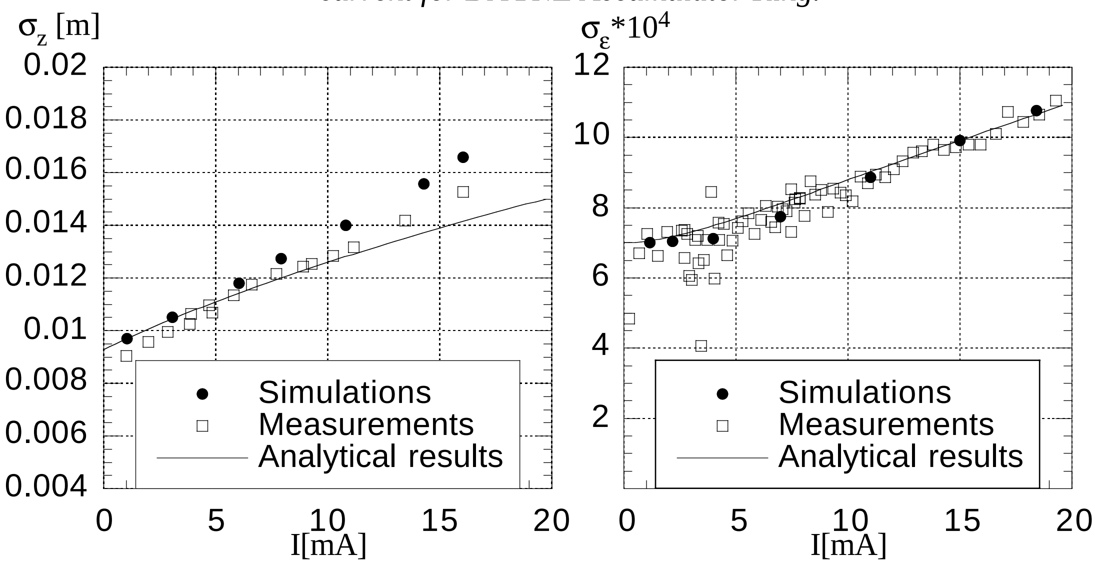

Figure 2: *Bunch length at* $\nu_s = 3.77 * 10^{-3}$ *and energy spread at* $\nu_s = 7.71 * 10^{-3}$ *vs current for ALS machine (by courtesy of J. Byrd).*

Also for ALS machine we have compared the results of equations (13) and (14) with both measurements and simulations, and the results are reported in figure 2 for two different synchrotron tunes obtained by changing RF peak voltage. The agreement is good for this machine too.

5 Conclusions

We have introduced the hypothesis that the wake fields could produce heating of the bunch thus increasing the energy spread even below microwave instability threshold. We have calculated the general expression for the energy variance induced by the wake fields, obtained the correspondent energy spread, and summed it quadratically to the natural energy spread.

The application to simple cases, pure resistive, pure inductive, and combined resistive - inductive wake fields, gives analytical expressions for bunch length and energy spread vs current, that can be compared with experimental data.

For DAΦNE Accumulator Ring and ALS the agreement with measurements is as good as that obtained with a time consuming simulation code. Moreover longitudinal mode coupling can still exist and produce a further increase of energy spread with a microwave instability threshold shifted to higher currents.

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