DAΦNE TECHNICAL NOTE

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Note: **RF-6**

A RF FEEDBACK FOR DAΦNE

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INTRODUCTION

In the RF-5 DAΦNE note we showed that is possible to avoid Sands and Robinson instabilities for any beam current up to 5 Amps by properly setting the RF system. This is in principle true, but the stability region around the working point may be too small, especially for high current values, getting the machine operation very much impractical and sensitive to any longitudinal transient oscillation.

There are several different techniques to reduce or compensate the beam loading effects on the center of mass stability and to broaden the stability region. A simple and reliable method consists in adding back a portion of the cavity voltage to the low level RF drive in the negative feedback scheme shown in Fig 1.

In this way the differential impedance seen by the beam, and then the beam induced voltage, is reduced by the open loop gain factor. This technique is called "RF feedback" or "Wide band feedback" and has been successfully tested in some proton machines, as the CERN SPS. In practice the performances are limited only by the total group delay τ of the overall loop that limits the maximum attainable feedback gain.

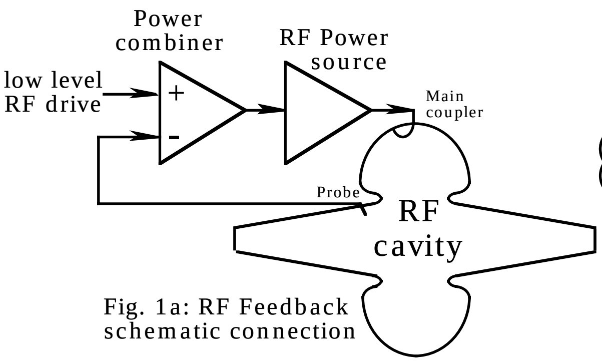

Fig. 1a: RF Feedback schematic connection

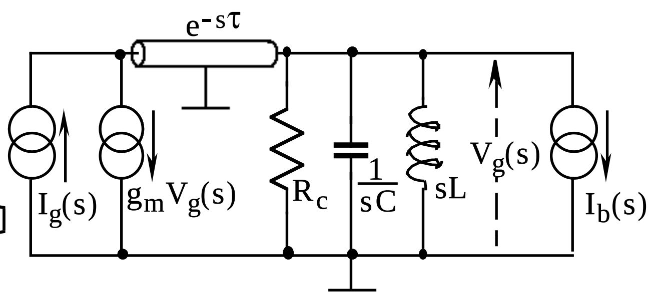

Fig. 1b: RF Feedback circuit (Laplace domain)

In this paper we compute the effectiveness of this method in the DAΦNE case and compare the results to those reported in RF-5 note.

1) THE STABILITY REGION

The values of the main RF parameters are listed in Tab. 1. Some of them show different values for the two considered cases of machine broadband impedance Z/n (1 or 2 Ohms).

		Z/n=1Ω		***Z/n=2Ω***
F_{RF}	RF Frequency (MHz)		368.25	
V_{gn}	Nominal Gap Voltage (KV)	130		260
R_S	Cavity Shunt Impedance (MΩ)		2.25	
Q_0	Cavity Unloaded Quality Factor		30.000	
V_r	Total Losses per turn (KV)	16.3		23.3
F_{si}	Incoherent Synchrotron (KHz) Frequency (@ V_{gn})	27.3		38.6
I_{dcM}	Max Average Stored Current (Amps)	5.24		2.62
N_b	Max Number of Bunches	120		60
β	Coupling Coefficient	25		5
P_{iM}	Max Available RF Power (KV)		150	

Tab. 1: RF parameter list.

The results of RF-5 note are reported in Fig. 2 in a more convenient way. The RF system working points are plotted on a plane having the beam loading factor Y on the vertical axis and the total load phase ϕ_L on the horizontal one. The beam loading factor Y is the ratio between the beam induced voltage and the actual gap voltage, and is defined as:

$$Y = \frac{2}{1+\beta} \frac{I_{dc} R_s}{V_g} \tag{1}$$

where β is the generator to cavity coupling factor, I_{dc} is the average beam current, R_s is the cavity shunt impedance and V_g is the cavity gap voltage.

The total load phase ϕ_L is the phase of the load seen by the RF generator, that includes the cavity impedance and the beam.

The shaded areas of Fig. 2 are the allowed regions for the working point in the ϕ_L-Y plane because they lie inside the Robinson, Sands and power limits.

The case Z/n=2Ω at nominal gap voltage V_g=260 KV is reported in Fig. 2a. The stable phase range is roughly 65° for 30 bunches (1.31 Amps, day-one goal) and 45° for 60 bunches (2.62 Amps). These values are further reduced if the gap voltage is lowered. They become respectively 60° and 35° at V_g=130 KV (see Fig. 2b).

The case $Z/n=1\Omega$ is considered in Fig. 2c at nominal gap voltage (130 KV). The stable phase range for 120 bunches (5.24 Amps, max machine current) is less than 30°. In the following we will compute how all these values are modified by a RF feedback scheme.

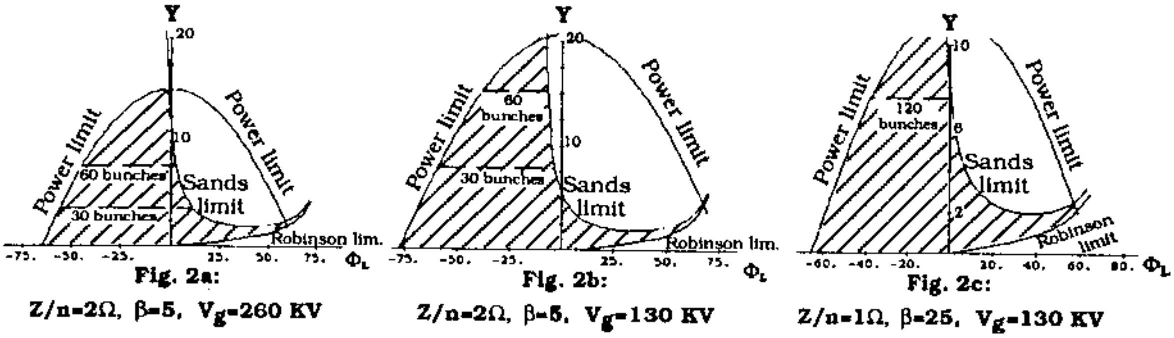

Fig. 2: Stability regions of the DAΦNE beam with a standard RF system

2) THE RF FEEDBACK

The synchrotron equation for an electron machine, including the beam to cavity accelerating mode interaction, is derived in Appendix and is of the following form:

$$\varepsilon + \frac{\omega_{si}^2 I_{dc} (Z_r^- - Z_r^+)}{\omega_{sc} V_g \sin \phi_s} \varepsilon + \omega_{si}^2 \left[1 + \frac{I_{dc} (Z_i^- + Z_i^+)}{V_g \sin \phi_s} \right] \varepsilon = 0 \tag{2}$$

where ε is the relative energy error of the circulating particle, ω_{sc} is 2π times the coherent synchrotron frequency, ϕ_s is the synchronous phase measured from the peak of the accelerating voltage, and $Z_r^{\pm}$, $Z_i^{\pm}$ are the real and imaginary parts of the impedance seen by the beam at $F_{RF} \pm F_{sc}$ frequencies.

The Sands limit is reached when the coherent synchrotron frequency approaches 0, that is:

$$\omega_{sc}^2 = \omega_{si}^2 \left[1 + \frac{I_{dc} (Z_i^- + Z_i^+)}{V_g \sin \phi_s} \right] = 0 \quad \Rightarrow \quad 1 + \frac{2\, I_{dc}\, Z_i(j\omega_{RF})}{V_g \sin \phi_s} = 0 \tag{3}$$

If there are no feedback connections, the impedance seen by the beam is just the resonant impedance of the cavity fundamental mode and is given by:

$$Z_{cav} = R_C \cos (\phi_Z)\, e^{j\phi_Z} \tag{4}$$

where ϕ_Z is the cavity tuning angle, i. e. the impedance phase at ω_{RF}, and $R_C = R_S/(1+\beta)$ is the loaded cavity shunt impedance. If ω_C is 2π times the cavity resonant frequency, ϕ_Z is related to the detuning $\Delta\omega = \omega_{RF} - \omega_C$ by the expression:

$$\tan(\phi_z) = -\frac{2\,Q_0\,\Delta\omega}{(1+\beta)\,\omega_C} = -\Delta\omega\,/\omega_{BW} \qquad (5)$$

where ω_{BW} is 2π times the half-bandwidth of the cavity.

This leads to the well known limit value of the Y variable given by:

$$Y_S = -\frac{2\sin\phi_s}{\sin(2\phi_z)} \qquad (6)$$

The RF feedback scheme modifies the impedance seen by the beam and then the instability threshold. Referring to Fig. 1b, which describes the feedback in the domain of the Laplace transform, we have:

$$V_g(s) = \frac{Z_{cav}(s)}{1+g_m\,Z_{cav}(s)\,e^{-s\tau}}\,[Ig(s) - Ib(s)] \qquad (7)$$

So the impedance seen by the beam in this case is:

$$Z(s) = \frac{Z_{cav}(s)}{1+H(s)} \qquad \Rightarrow \qquad H(s) = g_m\,Z_{cav}(s)\,e^{-s\tau} \qquad (8)$$

where H(s) is the open loop transfer function. In the frequency domain the function H is given by:

$$H(j\omega) = g_m\,R_c\cos[\phi_z(\omega)]\,e^{j\phi_z(\omega)}\,e^{-j\omega\tau} \qquad (9)$$

with the function $\phi_Z(\omega)$ defined in (5). To maximize the loop gain it is worth to trim the total group delay τ in such a way that $\omega_C\tau = 2n\pi$. In this case we have:

$$H(j\omega_C) = g_m\,R_c = A \qquad (10)$$

and A is the maximum value of the RF feedback transfer function H. Later on we will show how the total group delay τ limits the maximum attainable value of A.

Combining eqs. (3) and (8) under the condition (10), the Sands threshold in the RF feedback scheme becomes:

$$Y_f = -\frac{2\sin\phi_s}{\sin(2\phi_z)}\;\frac{1+\dfrac{A}{1+\tan^2(\phi_z)}\left[A+\dfrac{2\cos(\phi_z-\Delta\omega\,\tau)}{\cos(\phi_z)}\right]}{1+\dfrac{A\sin(\Delta\omega\,\tau)}{\tan(\phi_z)}} \qquad (11)$$

where ϕ_Z and $\Delta\omega$ are related by (5).

Before going into the details of (11) let us do some consideration on the restrictions posed by the total group delay on the maximum attainable feedback gain.

3) STABILITY CONDITIONS FOR THE RF FEEDBACK

The maximum open loop gain A must not exceed a certain threshold to avoid loop instability. We can use the standard Bode criterion to analyze the loop stability. If we set a $\pi/4$ phase margin, the frequency deviation $\Delta\omega_{0dB}$ corresponding to the 0 dB point is given by the eq.:

$$- \mathrm{atan}(\Delta\omega_{0dB}/\omega_{BW}) - \tau\,\Delta\omega_{0dB} = -\pi + \pi/4 \qquad (12)$$

that, with the approximation:

$$\mathrm{atan}(\Delta\omega_{0dB}/\omega_{BW}) = \pi/2 - \mathrm{atan}(\omega_{BW}/\Delta\omega_{0dB}) \approx \pi/2 - \omega_{BW}/\Delta\omega_{0dB}$$

gives a second order algebraic eq. to compute $\Delta\omega_{0dB}$.

This corresponds to a maximum loop gain given by (5) and (4):

$$A^2 = 1 + (\Delta\omega_{0dB}/\omega_{BW})^2 \qquad (13)$$

The resulting A value is a function of the total group delay τ and the cavity bandwidth ω_{BW}. In Tab. 2 are reported the A values for 3 possible τ values and for the two cases Z/n= 1 or 2 Ω.

	Z/n= 1 Ω (β = 25)	Z/n= 2 Ω (β = 5)
τ = 250 nsec	A = 4.2 = 12.5 dB	A = 14.8 = 23.4 dB
τ = 375 nsec	A = 3.1= 10.0 dB	A = 10.2 = 20.2 dB
τ = 500 nsec	A = 2.6 = 8.3 dB	A = 8.0 = 18.0 dB

Tab. 2: Max open loop gain under different conditions

4) CENTER OF MASS THRESHOLDS WITH RF FEEDBACK

The curves reported in Fig. 2 have been recalculated taking into account the RF feedback effects. We have considered 3 different values for the total group delay. The case τ = 250 nsec represents the shortest delay for a real equipment managing 150 KW. This value can be reached providing that fast tubes (tetrodes) instead of klystrons are used as RF final stages.

The case τ = 375 nsec represents a very good performances for a klystron-based equipment, while τ = 500 nsec seems to be a standard value.
We remind that the total load phase ϕ_L is related to the beam loading variable Y and the cavity phase ϕ_Z by the eq. :

$$\tan(\phi_Z) = \tan(\phi_L)\,[\,1 + Y\cos(\phi_S)\,] - Y\sin(\phi_S) \tag{15}$$

The new center of mass stability limits due to the RF feedback are reported in Fig. 3 together with the old limits of Fig. 2. The new curves have been computed by eliminating ϕ_Z between eqs. (11) and (15). The case $Z/n=2\Omega$ at nominal voltage 260 KV is reported in Fig. 3a. The shaded area represents the new stable region for the RF working point. We have considered the most conservative case τ = 500 nsec. Comparing Figs. 2a and 3a we can see that the stability margin is almost doubled at 30 and 60 bunches (respectively 115° and 90°) and, more important, it lies between only power limits that are not exactly instability thresholds. Nevertheless, a small portion of the ϕ_L - Y plane, weakly shaded in the Fig. 3a, is lost in the new stability limits.

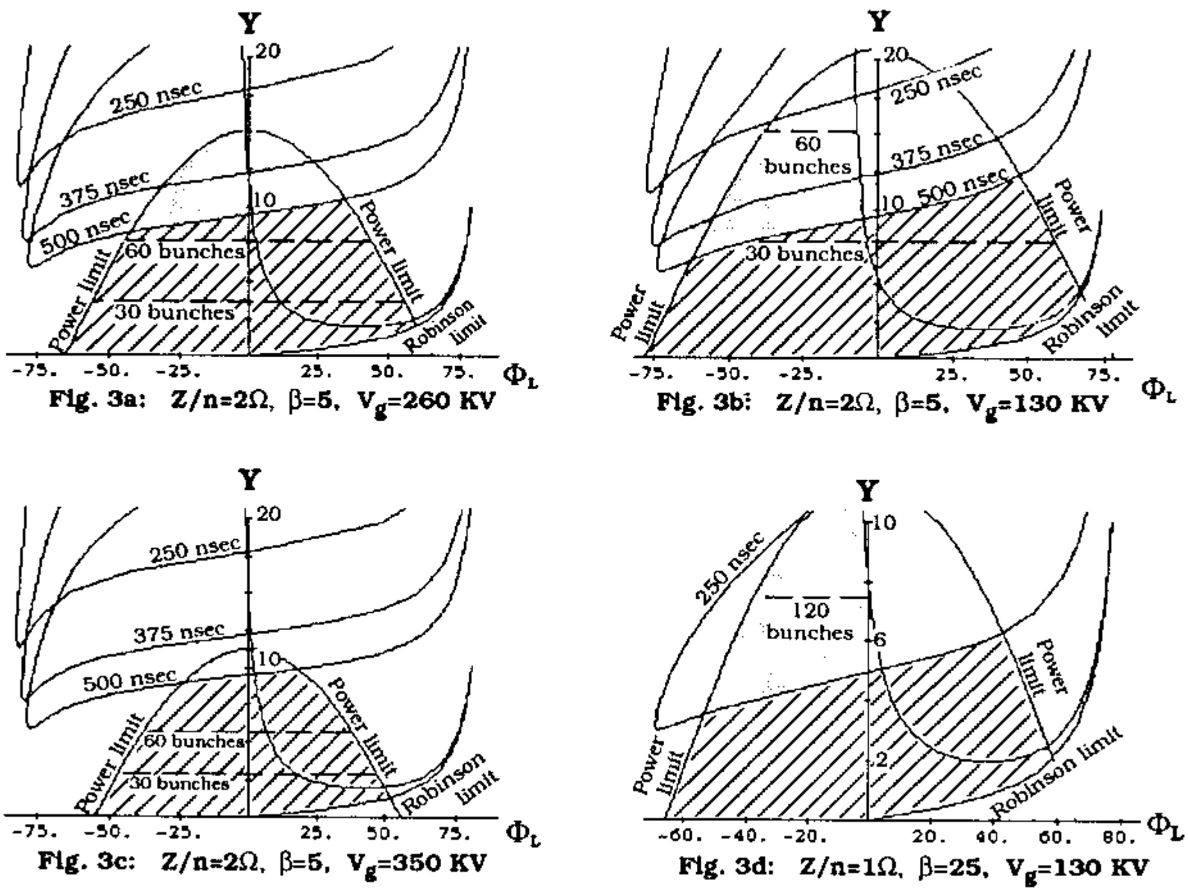

Fig. 3: Stability regions of the DAΦNE beam with a RF feedback.

This drawback is much more evident at lower voltage, as shown in Fig. 3b where the accelerating voltage has been reduced to 130 KV. In this case the stability margin for 30 bunches is always 115° but there is no more chance to store 60 bunches because the related Y value is far beyond the limit curve. This means that the RF feedback scheme is less flexible than standard RF arrangement for accelerating voltage variations. The situation is much more favorable if the voltage is larger than nominal value. A 350 KV value is considered in Fig. 3c; the stability region is again limited only by power considerations for 30 and 60 bunches.

The case $Z/n=1\Omega$ has been considered only in view of the highest machine current that is 5.24 Amps in 120 bunches. The stability diagram is reported in Fig. 3d at nominal voltage (130 KV). Due to the very high coupling factor $\beta=25$ the result in this case is awfully bad. The Y value corresponding to 120 bunches lays far beyond the limit curve even if we considered the most optimistic condition $\tau=250$ nsec.

5) CONCLUSIONS

The RF feedback scheme seems to be very effective for a bunch number up to 60, providing that the operating voltage is not sensitively lower than the nominal value. The case $Z/n=1\Omega$ is more critic but the β value may be lowered if an upper limit of 60 bunches is accepted.

On the contrary, as this RF feedback seems to be useless at 120 bunches, the equipment should be switched off near the top current value. To broaden the stability region even in this case a careful study of the possible improvements of the RF feedback (such some sophisticated phase compensating networks) is requested, or alternatively, a different beam loading compensation (ex. : feedforward scheme) has to be considered.

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APPENDIX

Since the results of this note are essentially based on eq. (2), it is worth to show its derivation. The derivation takes advantage of a perturbative method.

Let us consider a particle in its own longitudinal phase space (ϕ,ε), where ϕ is the particle phase delay respect to the synchronous phase ϕ_s, and $\varepsilon = \Delta E / E_0$ is the already mentioned relative energy error of the particle.

The equilibrium point in this phase space is then the origin point (0,0).

The energy gain of a particle over a machine turn may be generically expressed by:

$$\delta E(\phi,\varepsilon) = - U(\phi) - U'(\varepsilon) = - U(0) - \frac{\partial U}{\partial \phi}(0)\ \phi - U'(0) - \frac{\partial U'}{\partial \varepsilon}(0)\ \varepsilon \qquad (1a)$$

where $U(\phi)$ and $U'(\varepsilon)$ give the dissipated energy as functions of the phase and energy deviations. The synchronous particle neither gain nor lose any energy. This means:

$$U(0) + U'(0) = 0$$

So, dividing eq. (1a) by the revolution time T_0 and the machine energy E_0 we obtain:

$$\dot{\varepsilon} = - \frac{\omega_0}{2\pi E_0}\ \frac{\partial U'}{\partial \varepsilon}(0)\ \varepsilon - \frac{\omega_0}{2\pi E_0}\ \frac{\partial U}{\partial \phi}(0)\ \phi \qquad (2a)$$

where $\omega_0 = \omega_{RF}/h$ is 2π times the particle revolution frequency. Tacking into account the lattice dispersion:

$$\dot{\phi} = \alpha\ \omega_{RF}\ \varepsilon \qquad (3a)$$

where α is the machine momentum compaction, the time derivative of eq. (3a) leads to:

$$\ddot{\varepsilon} + \frac{\omega_0}{2\pi E_0}\ \frac{\partial U'}{\partial \varepsilon}(0)\ \dot{\varepsilon} + \frac{h\ \alpha\ \omega_0^{\ 2}}{2\pi E_0}\ \frac{\partial U}{\partial \phi}(0)\ \varepsilon = 0 \qquad (4a)$$

which is a generic form of the synchrotron equation. The functions U and U' are the potential for the elastic and frictional forces involved in the motion. We use a perturbative approach [4] to derive them.

Let us consider a beam oscillating longitudinally in the rigid mode. Let us assume a beam current phase of the form:

$$\phi(t) = \phi_M \cos(\omega_{sc}\ t) \qquad (5a)$$

where ω_{sc} is the coherent synchrotron frequency and ϕ_M is the amplitude of the phase oscillations.

The beam current at RF harmonic may be written as:

$$I(t) = 2\ I_{dc} \cos[\omega_{RF}\ t - \ \phi_M \cos(\omega_{sc}\ t)] \qquad (6a)$$

If ϕ_M is small enough, eq. (6a) becomes:

$$I(t) \approx 2I_{dc} \cos (\omega_{RF}\ t) + I_{dc}\phi_M\ [\sin(\omega_{RF} + \omega_{sc})t + \sin(\omega_{RF} - \omega_{sc})t] \qquad (7a)$$

The 3 frequency components of the beam current, together with the RF generator, interact with the cavity impedance and give rise to 3 voltages. The carrier component of the gap voltage is opposite in phase and anticipates by ϕ_s with respect to the carrier of the beam current. So the total voltage is given by:

$$\begin{aligned} V(t) = &- V_g \cos (\omega_{RF}\ t+\phi_s)+I_{dc}\phi_M\ Z^+ \sin[(\omega_{RF} + \omega_{sc})t+ \phi^+] + \\ &+ I_{dc}\phi_M\ Z^- \ \sin[(\omega_{RF} - \omega_{sc})t+ \phi^-] \end{aligned} \qquad (8a)$$

where $Z^{\pm}$ and $\phi^{\pm}$ are the modules and phases of the cavity impedance at the two sideband frequencies.

The instantaneous dissipated power P(t) is the product of the two functions I(t) and V(t). By neglecting the RF components of P(t) (thay average out over a fraction of turn) and the terms containing $\phi_M{}^2$ we get:

$$\begin{aligned} P(t) = &- V_gI_{dc} \cos(\phi_s) + I_{dc}{}^2\phi_M\ [Z^+ \sin(\omega_{sc}t+ \phi^+) - Z^- \sin(\omega_{sc}t- \phi^-)] - \\ &- (V_gI_{dc}\phi_M\ /2)\,[\sin(\ \omega_{sc}t- \phi_s) - \sin(\omega_{sc}t+ \phi_s)] = \\ &= - V_gI_{dc} \cos (\phi_s) + [\phi(t)\ I_{dc}{}^2/\omega_{sc}]\ (Z_r{}^- - Z_r{}^+) \\ &+ \phi(t)\ I_{dc}{}^2(Z_i{}^-+Z_i{}^+) + V_gI_{dc}\phi(t) \sin(\phi_s) \end{aligned} \qquad (9a)$$

The energy gained by each particle in one turn is then:

$$\begin{aligned} \delta E = - P(t)T_0\ /N = \ &qV_g \cos(\phi_s) - [qI_{dc}\phi(t)/\omega_{sc}]\ (Z_r{}^- - Z_r{}^+) - \\ &- qI_{dc}\phi(t)\ (Z_i{}^-+Z_i{}^+) - qV_g\phi(t) \sin(\phi_s) \end{aligned} \qquad (10a)$$

where N is the number of particles and q is the electron charge. By comparing (10a) to (1a) and reminding (3a) we finally obtain:

$$\frac{\partial U'}{\partial \varepsilon} = \frac{qI_{dc}\alpha\ \omega_{RF}}{\omega_{sc}}\ (Z_r{}^- - Z_r{}^+)\ \ ;\ \ \frac{\partial U}{\partial \phi} = q\ I_{dc}\ (Z_i{}^- + Z_i{}^+) + qV_g \sin(\phi_s) \qquad (11a)$$

The incoherent synchrotron frequency $F_{si} = \omega_{si}\ /2\pi$ is given by the well-known relation:

$$\omega_{si}{}^2 = \frac{h\ \omega_0{}^2\ \alpha\ q\ V_g \sin(\phi_s)}{2\ \pi\ E_0} \qquad (12a)$$

Combining (4a) and (11a), and tacking into account (12a) we finally obtain eq. (2).