DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, April 22, 1997

Note: **MM-24**

FIELD QUALITY OF THE LARGE QUADRUPOLES FOR THE DAFNE MAIN RINGS

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1. Introduction

The field quality of 28 "large" quadrupoles has been measured to check if each magnet meets the Specifications and to obtain the high order harmonic distributions for beam tracking simulations. The measurements on the prototype are described in [1].

The measurements have been performed by means of the DANFYSIK rotating coil system [3]. The integrated gradient, the average value of the field deviation from the quadrupole component at the boundary of the good field region (30 mm from the magnet axis) and the contribution of each individual high order component have been measured at 11 different excitation currents (corresponding to 10%, 20%.....,100% of the maximum current plus a "nominal" working point set in the Specification).

The position of the magnetic center has also been recorded with respect to the reference optical devices placed on top of the magnets. The mechanical center positions have been measured by the Alignment Group. The comparison between the results obtained by the two groups will be described in a Technical Note together with those concerning the large sextupoles as soon as all data will be available.

This Note contains all the measured values for the 28 quadrupoles. A discussion of the results is given in the text, together with the most significant plots and tables. The measured values for each magnet at the above mentioned excitation currents are given in the Appendix, where each magnet is identified by its serial number (the prototype is indicated as Serial#0). Although the measurement have been performed up to ≈ 9 T/m, the gradients on the nominal working points of the DAΦNE Main Rings at 0.51 GeV never exceed ≈ 5 T/m. For sake of simplicity, we show therefore the distributions at an excitation current of ≈ 90A, and, when useful, at the maximum current, where there is a small amount of saturation.

2. Integrated gradient

The measurements have been performed by setting the power supplies always at the same nominal values, and the current detected by means of the precision DCCT system of the DANFYSIK system. The measured currents are constant within few mA. We recall that each quadrupole in the Main Rings has its own power supply and can therefore be calibrated individually. However, it is interesting to show how the integrated gradients are distributed around their average values.

Figure 1 shows the integrated gradient, averaged over the sample of the 28 magnets, as a function of the excitation current. Since the r.m.s. width of the distribution is too small to be seen in the figure, it is given in Table I.

Figure 2 shows the distribution of the measured gradients at 90 A (the magnetic length of the magnets is ≈ 30 cm [1]). The measured values for each magnet at all excitation currents are given in Table A1 in the Appendix.

TABLE I - Integrated gradients averages and distribution widths

Current (A)	$<\int Gdl$ (T)$>$	$\int Gdl$ (T) r.m.s.
18.102 ± 004	0.30093	0.00033
36.194 ± 004	0.59877	0.00059
54.282 ± 004	0.89712	0.00096
64.654 ± 004	1.06780	0.00104
72.364 ± 004	1.19450	0.00122
90.453 ± 004	1.49030	0.00148
108.540 ± 004	1.78410	0.00185
126.630 ± 004	2.07300	0.00211
145.940 ± 004	2.35720	0.00261
162.750 ± 004	2.54830	0.00315
180.800 ± 004	2.72040	0.00361

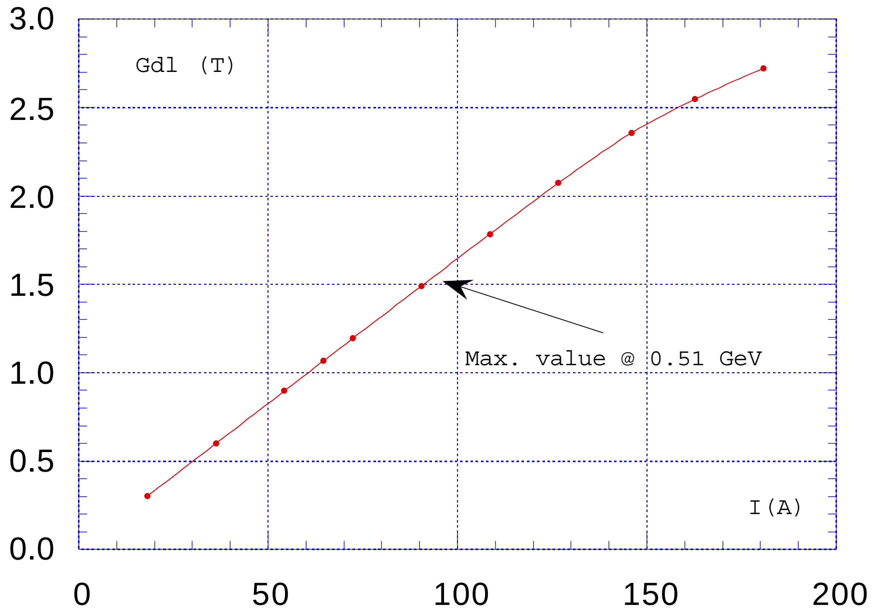

Fig. 1 - Average integrated gradient versus excitation current.

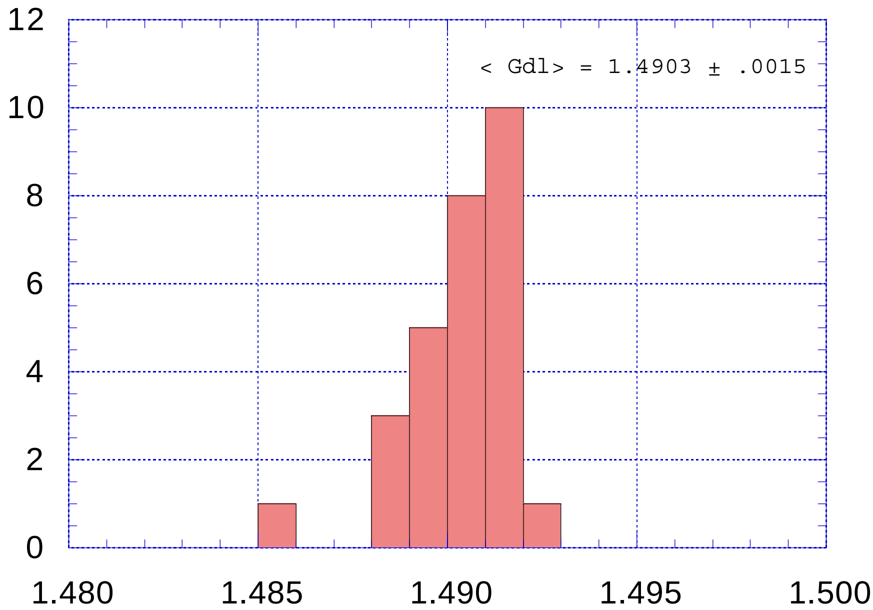

Fig. 2 - Integrated gradient distribution (90.45 A).

2. Average deviation from the ideal field

The Specification for the field quality has been set as ± 5×10^{-4} up to an excitation current of 146.5 A at the boundary of the good field region (a circle of 30 mm radius around the magnet axis). The measurements, however, have been performed up to 180.8 A, and the last two currents are above the specified maximum . Of course, the rotating coil system checks the integrated field only. In order to verify that the Specification is met for all the magnets, the output of the rotating coil system has been used as an input deck for a computer code, whose output lists the field contribution of each harmonic component at 30 mm from the axis, its phase and the ratio between the absolute value of the component and the main quadrupole one, always at 30 mm. In addition, the code sums up all high order radial components, as well as the azimuthal ones, and takes their vector sum, divided by the main quadrupole component, in steps of two degrees along the azimuthal coordinate. The result is the overall deviation from the ideal field, and it is given as a plot, like the one shown in Fig. 3.

There are two interpretations on the check that the magnets meet the Specification: the first one is to require that the average value of the full line in Fig. 3 (|B|) on the circle of 30 mm radius be less than 5×10^{-4}. This average is given for each quadrupole and each excitation current in Table A2, from which it can be seen that all the magnets are within the specified limit (as mentioned before, the last two columns are above the specified maximum current). A more restrictive interpretation of the Specification is that the 5×10^{-4} limit on |B| be not exceeded at any azimuth in Fig. 3. In this case 5 quadrupoles exceed the limit, as shown in Table II. The distributions of the average deviation among the quads at 90.45 A and 180.80A are shown in Figs 4 and 5 respectively.

TABLE II - Quadrupoles exceeding the limit of $\Delta B/B = 5 \times 10^{-4}$ @ 30 mm
at any point of the good field region boundary (units of 10^{-4})

Current (A)	Serial#0	Serial#1	Serial#8	Serial#23	Serial#27A
18.102			5.8	5.5	
36.194			6.0	5.9	
54.282			6.2	5.5	
64.654			6.2	5.9	
72.364			6.3	5.9	
90.453			6.4	5.9	
108.540			6.3	5.5	
126.630	5.2		6.5	6.0	5.2
145.940		5.5	5.8	5.8	

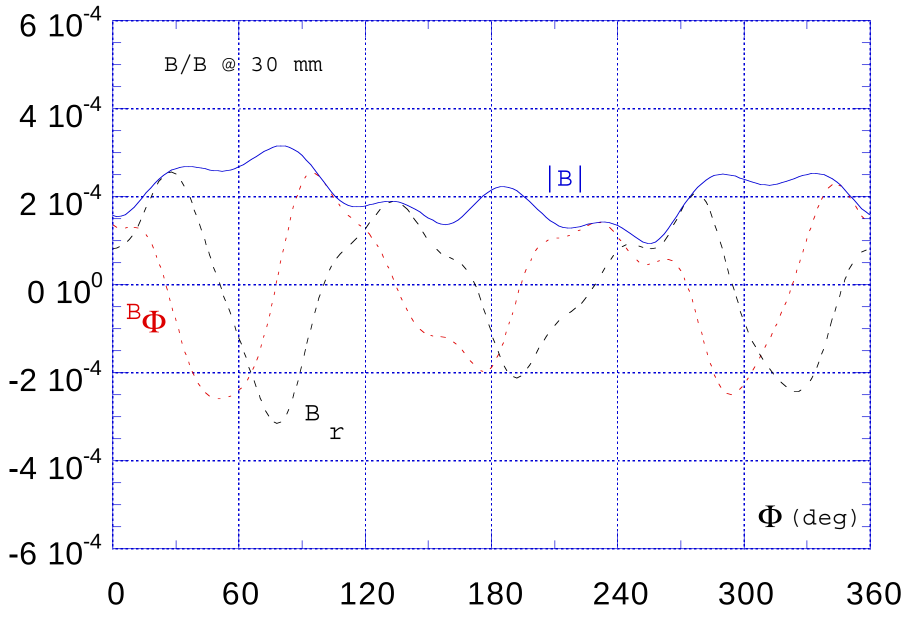

Fig. 3 - Field quality of Quadrupole Serial #10 at 90.45A.

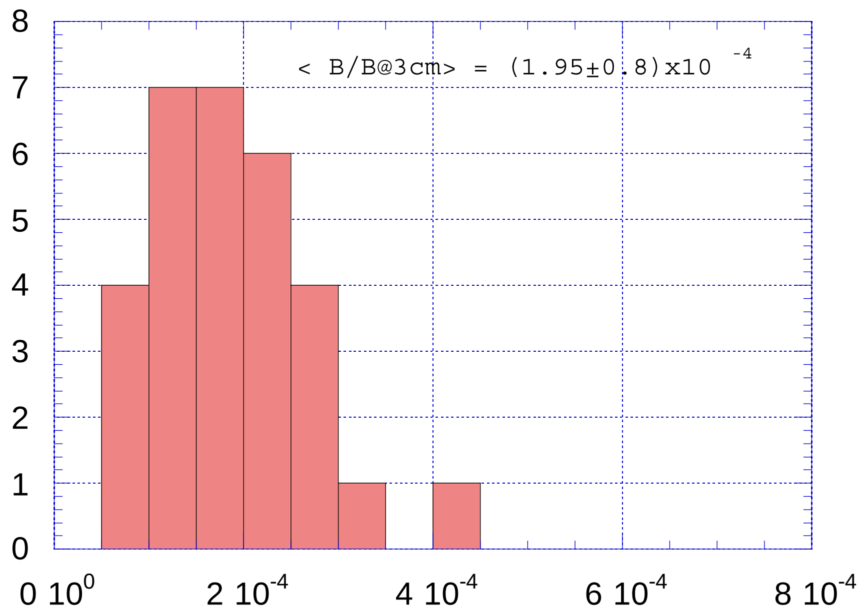

Fig. 4 - Distribution of the average field error at 30 mm (90.45A)

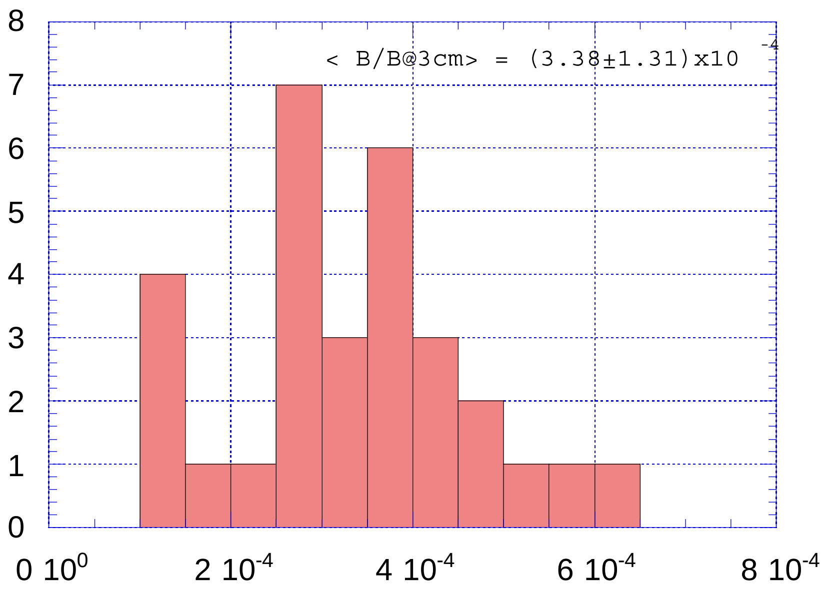

Fig. 5 - Distribution of the average field error at 30 mm (180.80A)

As it can be observed from Table II, the specified limit of 5×10^{-4} is preferably exceeded at high excitation current, where the magnets begins to saturate. As explained in detail in the next paragraph, this contribution comes mainly from the sextupole component, which can be split in two parts: the first one depends on the magnet shape, and can be observed also at low excitation currents, while the second one comes from saturation, and becomes dominant at high currents. For some quads these two contributions are in phase, and for some others they tend to cancel out, and this explains why for only two quads the limit is exceeded at all currents while in most others this happens only at high current. The behaviour of the average deviation from the ideal field at the boundary of the good field region, averaged over all the quadrupole sample, is shown in Fig. 6 versus the excitation current, the error bars being the r.m.s. widths (the distributions are not Gaussian).

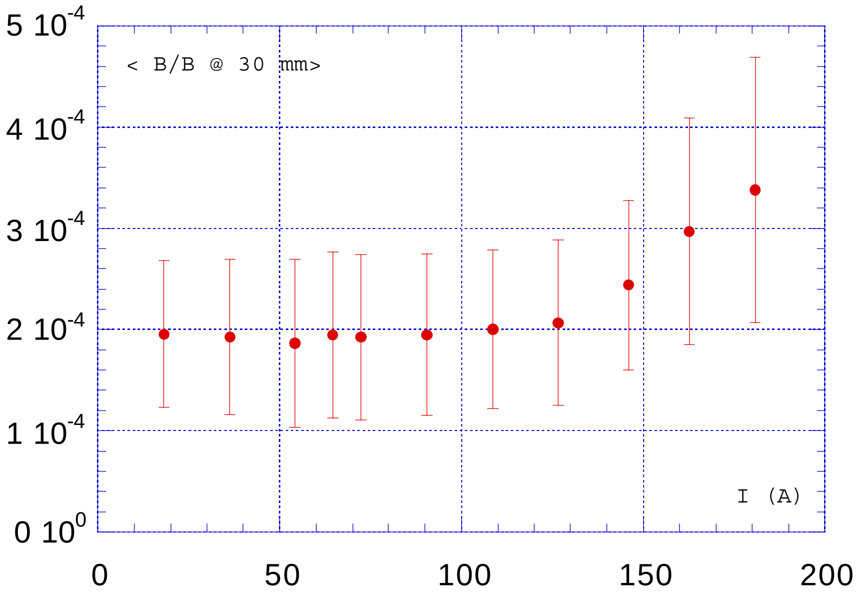

Fig. 6 - Average deviation from the ideal field @ 30 mm averaged over the 28 quads (error bars are the r.m.s. deviations of the non-Gaussian distributions)

3. Sextupole term

The contribution of the sextupole term to the overall deviation from the ideal field at 30 mm from the axis is given in Table A3. We recall that the sensitivity of the measuring system to the sextupole contribution is of the order of 10^{-4} [2]. Figure 7 shows the distribution of the sextupole term at 90.45A, Fig. 8 is the same at the maximum current of 180A, while Fig. 9 gives the average value and the r.m.s. width of the distribution as a function of the excitation current. From the comparison of Figs 6 and 9 it appears clearly that most of the field distortion comes from the sextupole term. The phase is distributed at random.

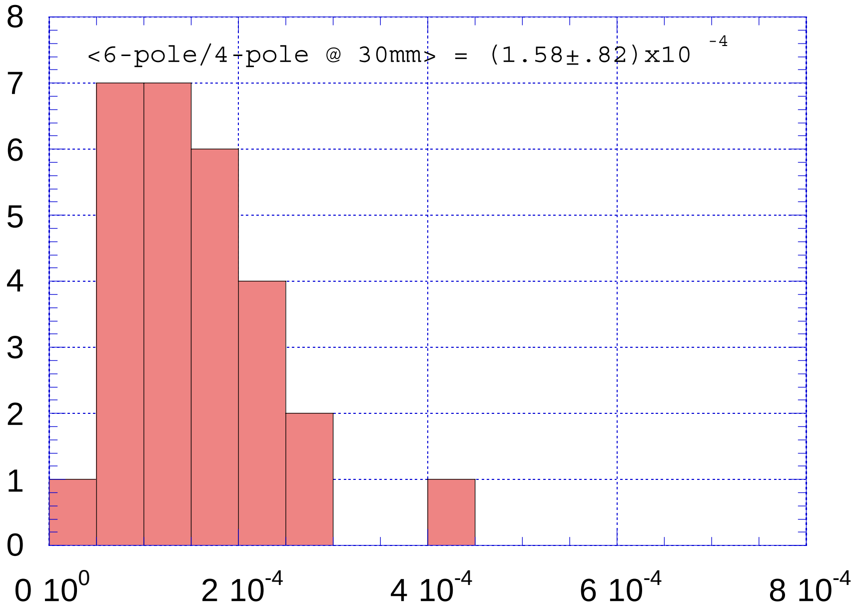

Fig. 7 - Distribution of sextupole/quadrupole @ 30 mm (90.45A)

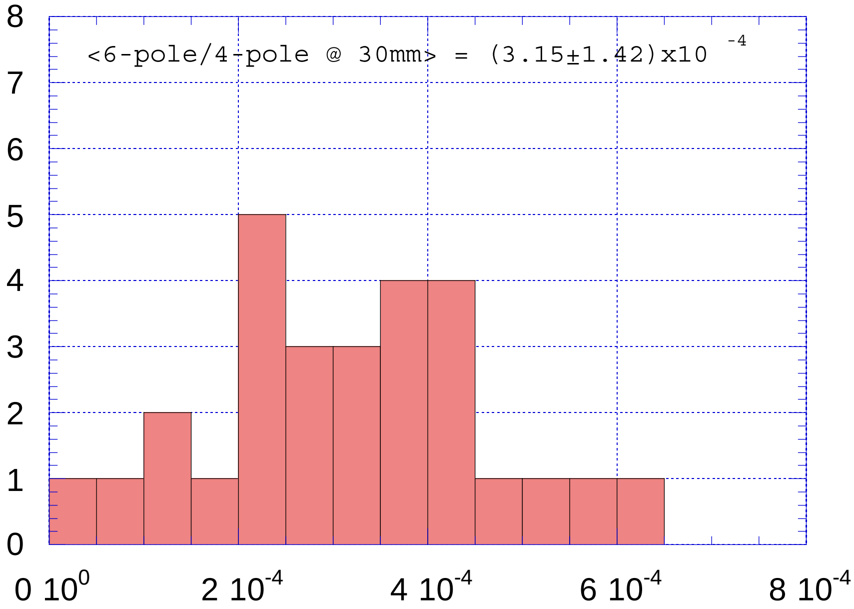

Fig. 8 - Distribution of sextupole/quadrupole @ 30 mm (180.80A)

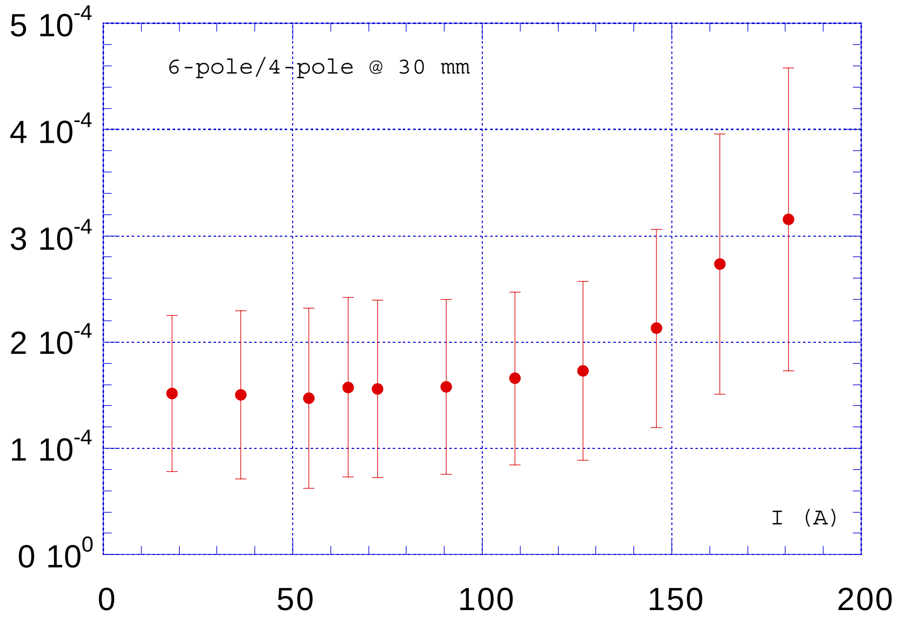

Fig. 9 - Sextupole/quadrupole @ 30 mm averaged over the 28 quads
(error bars are the r.m.s. deviations of the non-Gaussian distributions)

4. Octupole and decapole terms

The contribution of the octupole term to the overall deviation is much smaller than the sextupole one; however, it is larger than the result found for the small quadrupoles [2] and significantly larger than the expected sensitivity. The values are given in Table A4, while the distribution at 90.45 A is shown in Fig. 10. Figure 11 shows the behaviour of the octupole contribution, averaged over all the quads, versus the excitation current. The phase is distributed at random.

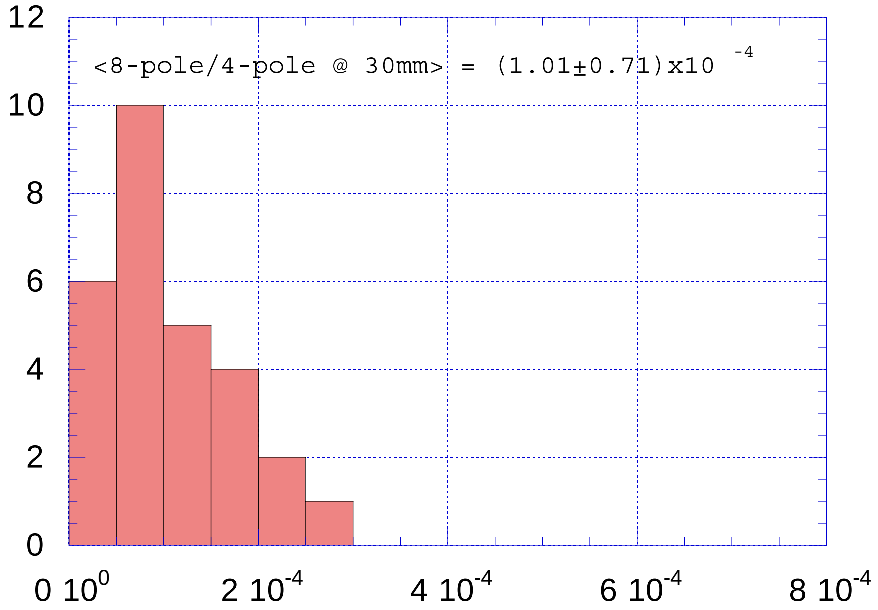

Fig. 10 - Distribution of octupole/quadrupole @ 30 mm (90.45A)

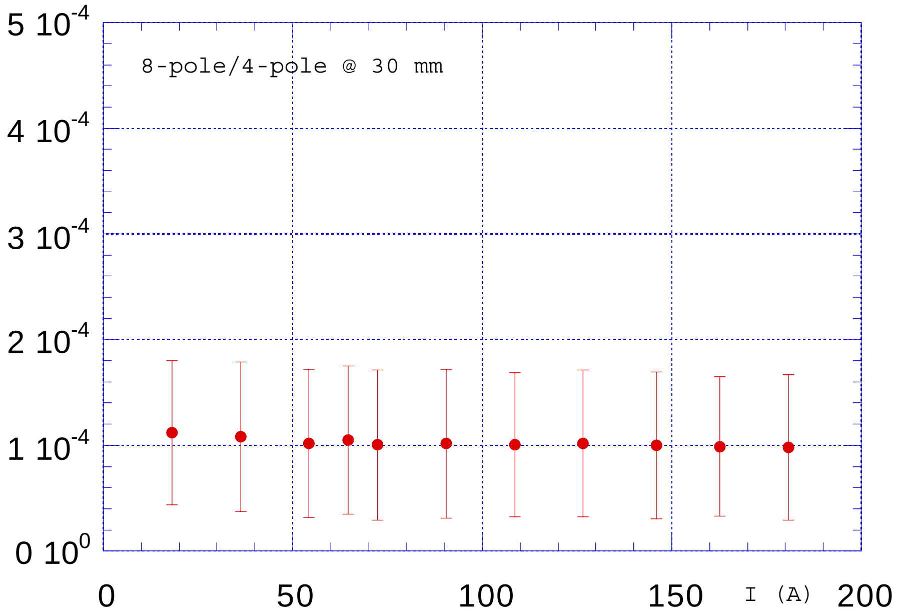

Fig. 11 - Octupole/quadrupole @ 30 mm averaged over the 28 quads
(error bars are the r.m.s. deviations of the non-Gaussian distributions)

The contribution of the decapole term is smaller, of the order of the expected sensitivity. Averaging over all quadrupoles and excitation currents we find (1.1±0.7)x10^{-5}, which is negligible with respect to the other contributions.

5. Twelve-pole term

The 12-pole term is a systematic high order component, with the same symmetry of the main quadrupole term. Its contribution to the overall deviation from the ideal field has been minimised by chamfering the end caps of the poles in the prototype magnet [1].

The residual contribution to the field deviation is very small, and increases by a very small amount at high current, as shown in Fig. 12 (the vertical scale is 5 times smaller than in the previous figures).

The systematic nature of this high order harmonic appears clearly in the same figure from the small error bars around the average points, which represent the r.m.s. width of the distributions, in this case similar to Gaussian (see Fig. 13).The slight increase of both the average and r.m.s. width of the distribution at low current is probably due to the reduced sensitivity of the measuring system at such small signal levels. The phase of the harmonic is opposite to the main quadrupole one. The measured values for all the quads are given in Table A5.

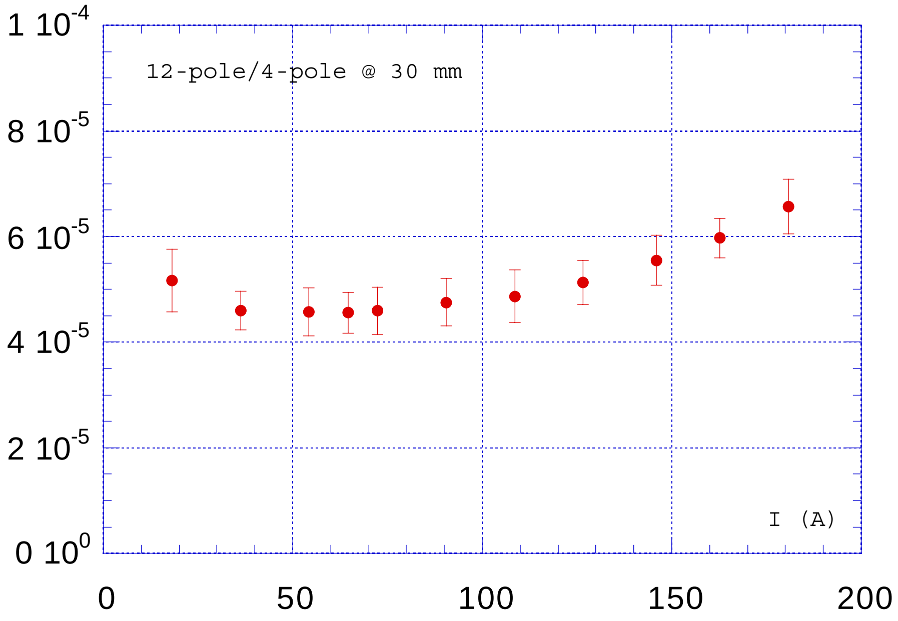

Fig. 12 - Twelve-pole/quadrupole @ 30 mm averaged over the 28 quads
(error bars are the r.m.s. deviations of the Gaussian-like distributions)

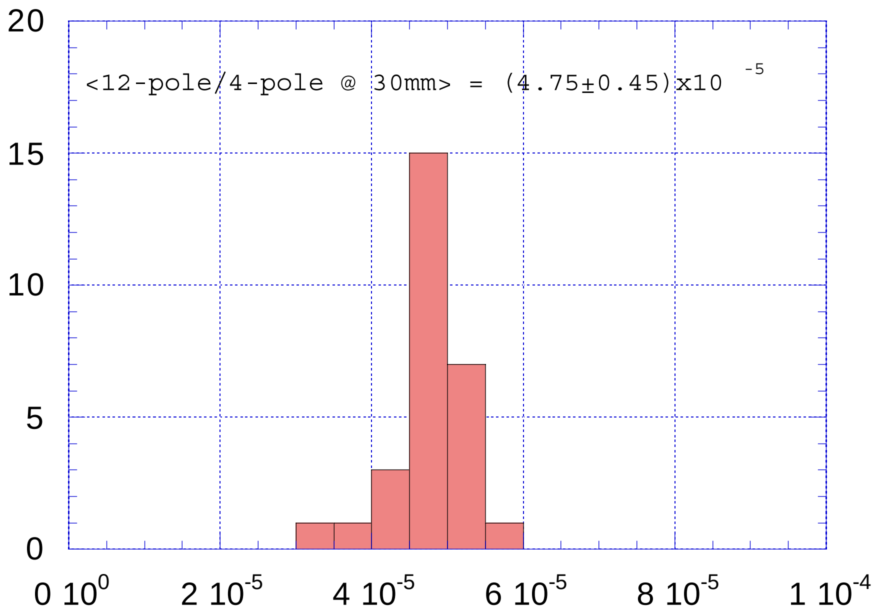

Fig. 13 - Distribution of 12-pole/quadrupole @ 30 mm (90.45A)

6. Twenty-pole term

As well as the 12-pole term, also the 20-pole is a systematic high order harmonic with the same symmetry of the main quadrupole component. In the large quadrupoles this component is very small ($\approx 3x10^{-5}$), but still within the sensitivity of the measurement system, due to the high order of the harmonic.

Figure 14 shows the 20-pole component, averaged over the sample of the 28 magnets, versus the excitation current on a vertical scale even smaller than in the plot dedicated to the 12-pole, in order to have a visible size of the distribution widths. Also in this case the distribution is Gaussian-like (see Fig. 15). The phase is always opposite to the main quadrupole one. The values for the single magnets are given in Table A6.

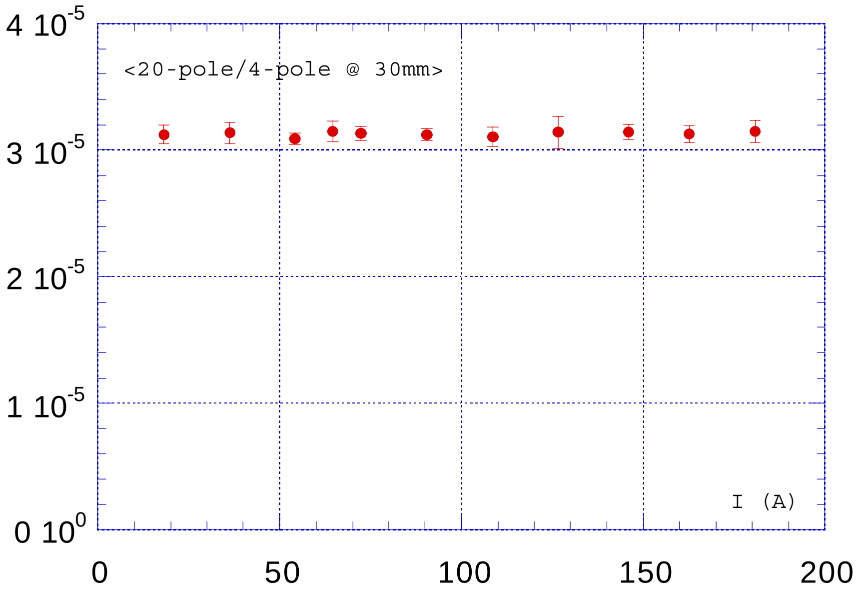

Fig. 14 - Twelve-pole/quadrupole @ 30 mm averaged over the 28 quads
(error bars are the r.m.s. deviations of the Gaussian-like distributions)

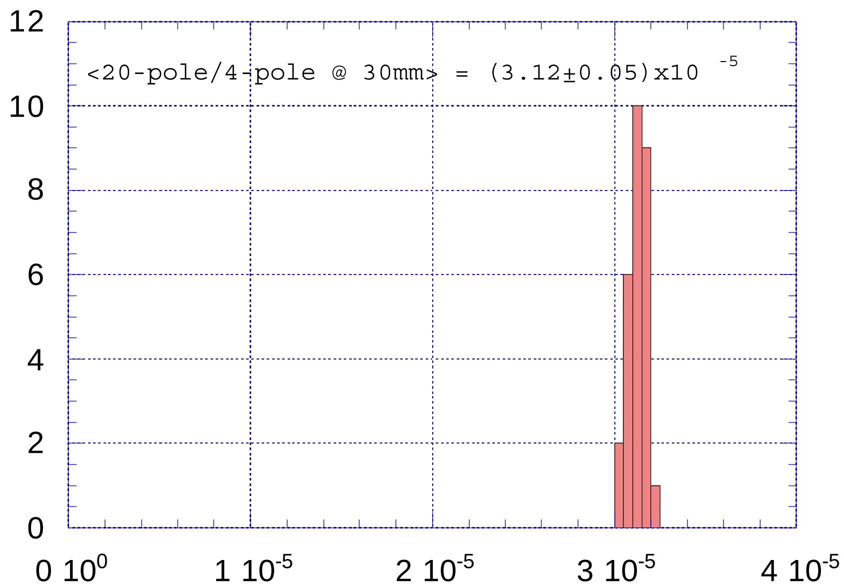

Fig. 15 - Distribution of 20-pole/quadrupole @ 30 mm (90.45A)

7. Conclusions

The Specification on the large Main Ring quadrupole field quality is practically always met at the gradients foreseen on the nominal operating points of DAΦNE. Table A2 can be used to arrange the magnets in the rings in such a way that those quads where the tolerance is exceeded by the largest amount are employed where low gradients are required.

Although from the results reported in this paper it is possible to simulate the real machine with errors, it seems more practical to assume the average values of the high order multipoles as systematic errors and the widths of the distribution as random ones. Table III summarizes these values (averaged over all excitation currents up to 90.45A), to be used in a simulation tracking program.

TABLE III - Systematic and random multipoles
(expressed as fraction of main quadrupole field at 30 mm, units of 10^{-4})

Multipole	Systematic	Phase	Random (rms)	Phase
6-pole	1.5	random	0.8	random
8-pole	1.0	random	0.7	random
12-pole	0.47	π	0.05	π
20-pole	0.31	π	0.01	π

References

[1] B. Bolli, F. Iungo, N. Ganlin, M. Paris, M. Preger, C. Sanelli, F. Sardone, F. Sgamma, M. Troiani: "Misura delle caratteristiche magnetiche del quadrupolo prototipo per gli archi dei Main Rings - DAΦNE Technical Note MM-13 (19/4/1996).

[2] B. Bolli, N. Ganlin, F. Iungo, F. Losciale, M. Preger, C. Sanelli, F. Sardone: "Field quality of the small quadrupoles for the DAΦNE Main Rings" - DAΦNE Technical Note MM-10 (22/2/1996).

[3] F. Iungo, M. Modena, Q. Qiao, C. Sanelli: "DAΦNE Magnetic Measurements Sustem" - DAΦNE Technical Note MM-1 (4/11/1993).

APPENDIX

Table A1 - Integrated gradients (T) of the 28 large quadrupoles

Serial #	18.10 A	36.19 A	54.28 A	64.65 A	72.36 A	90.45 A	108.54 A	126.63 A	145.94 A	162.75 A	180.80 A
0	0.29972	0.59663	0.89397	1.06465	1.19032	1.48537	1.77827	2.06651	2.35067	2.54349	2.71616
1	0.30047	0.59843	0.89581	1.06626	1.19247	1.48827	1.78172	2.07081	2.35648	2.54901	2.72163
2	0.30117	0.59911	0.89799	1.06852	1.19525	1.49140	1.78560	2.07519	2.35897	2.54969	2.72079
3	0.30083	0.59839	0.89672	1.06732	1.19380	1.48963	1.78339	2.07251	2.35921	2.55023	2.72245
4	0.30101	0.59893	0.89736	1.06822	1.19539	1.49064	1.78450	2.07385	2.36009	2.55273	2.72545
5	0.30114	0.59911	0.89744		1.19486	1.49091	1.78472	2.07371	2.35874	2.55112	2.72461
6	0.30110	0.59918	0.89777	1.06858	1.19529	1.49145	1.78545	2.07490	2.35929	2.55021	2.72245
7	0.30117	0.59928	0.89782	1.06863	1.19544	1.49152	1.78542	2.07481	2.36115	2.55299	2.72483
8	0.30072	0.59832	0.89637	1.06691	1.19423	1.48906	1.78233	2.07074	2.35370	2.54399	2.71536
9	0.30075	0.59839	0.89641	1.06699	1.19502	1.48935	1.78298	2.07200	2.35606	2.54831	2.72193
10	0.30088	0.59880	0.89741	1.06868	1.19534	1.49055	1.78427	2.07348	2.35693	2.54757	2.71916
11	0.30074	0.59846	0.89675	1.06748	1.19403	1.49016	1.78382	2.07247	2.35481	2.54607	2.71838
12	0.30049	0.59793	0.89577	1.06616	1.19264	1.48819	1.78161	2.07031	2.35529	2.54695	2.71960
13	0.30114	0.59922	0.89793	1.06869	1.19546	1.49175	1.78586	2.07508	2.35673	2.54720	2.71980
14	0.30132	0.59953	0.89821	1.06909	1.19593	1.49201	1.78610	2.07544	2.35885	2.54946	2.72148
15	0.30115	0.59920	0.89797	1.06862	1.19543	1.49144	1.78655	2.07462	2.35947	2.55068	2.72257
16	0.30130	0.59936	0.89787	1.06864	1.19533	1.49137	1.78530	2.07439	2.35743	2.54797	2.71998
17	0.30100	0.59895	0.89738	1.06815	1.19480	1.49088	1.78548	2.07406	2.35995	2.55277	2.72534
18	0.30121	0.59939	0.89874	1.06886	1.19560	1.49191	1.78602	2.07556	2.36031	2.55221	2.72506
19	0.30100	0.59863	0.89694	1.06765	1.19429	1.49013	1.78469	2.07288	2.35775	2.54845	2.72001
20	0.30066	0.59817	0.89611	1.06656	1.19297	1.48823	1.78127	2.06949	2.35147	2.54014	2.71066
21	0.30089	0.59888	0.89724	1.06845	1.19453	1.49050	1.78409	2.07300	2.35572	2.54593	2.71752
22	0.30080	0.59845	0.89653	1.06705	1.19351	1.48916	1.78250	2.07079	2.35443	2.54324	2.71410
23	0.30108	0.59909	0.89759	1.06835	1.19503	1.49120	1.78510	2.07434	2.35828	2.54946	2.72177
24	0.30091	0.59873	0.89706	1.06760	1.19430	1.49018	1.78402	2.07259	2.35712	2.54957	2.72314
25	0.30094	0.59866	0.89679	1.06757	1.19399	1.48976	1.78325	2.07192	2.35505	2.54488	2.71593
26	0.30120	0.59926	0.89786	1.06848	1.19516	1.49129	1.78515	2.07435	2.35911	2.55011	2.72181
27A	0.30122	0.59908	0.89755	1.06839	1.19503	1.49111	1.78512	2.07426	2.35853	2.54864	2.71977

Table A2 - Average deviation from the ideal quadrupole field
on a circle of 30 mm radius around the magnet axis (units of 10^{-4})

Serial#	18.10A	36.19A	54.28A	64.65A	72.36A	90.45A	108.54A	126.63A	145.94A	162.75A	180.80A
0	2.78	2.88	2.88	2.81	2.90	2.76	2.79	2.89	2.63	2.52	2.76
1	2.86	2.56	2.65	2.81	2.69	2.80	2.63	2.83	4.18	**5.30**	**5.50**
2	1.51	1.16	1.15	1.35	1.31	1.58	1.52	1.61	1.62	1.86	2.13
3	1.31	1.49	1.52	1.39	1.62	1.53	1.61	1.71	2.72	3.40	3.81
4	2.28	2.04	2.12	2.31	2.34	2.18	2.39	2.63	3.33	3.96	4.36
5	0.92	1.23	1.14		1.17	1.44	1.28	1.53	1.55	2.36	3.14
6	1.79	2.41	2.26	2.41	2.48	2.69	2.80	3.08	3.16	3.89	4.44
7	1.69	1.44	1.36	1.39	1.41	1.42	1.44	1.54	1.81	2.17	2.69
8	3.81	4.11	4.30	4.34	4.41	4.37	4.34	4.39	3.78	3.40	3.03
9	1.08	1.14	1.28	1.22	1.42	1.24	1.53	1.18	1.68	2.45	2.77
10	1.84	2.16	2.15	2.04	2.01	2.05	2.09	2.04	2.83	3.56	3.75
11	1.16	1.51	1.22	1.24	1.33	1.12	1.32	1.51	2.87	4.36	**5.61**
12	1.53	1.49	1.49	1.41	1.31	1.46	1.50	1.54	1.46	1.46	1.80
13	2.29	2.62	2.60	2.53	2.50	2.50	2.50	2.39	2.14	2.46	2.69
14	1.92	2.13	2.29	2.27	2.12	2.19	2.37	2.51	3.29	4.96	**6.25**
15	2.36	2.32	1.74	1.98	1.73	1.76	1.77	1.85	3.34	4.51	4.60
16	0.99	1.27	1.19	1.21	1.22	1.31	1.20	1.26	1.66	1.42	1.23
17	1.20	1.06	0.69	0.80	1.08	0.98	1.19	1.41	1.18	0.85	1.01
18	1.03	0.80	0.79	0.77	0.80	0.82	0.88	0.94	1.19	2.07	2.78
19	2.11	1.66	1.74	1.79	1.45	1.44	1.96	1.62	2.02	3.04	3.70
20	1.25	0.99	1.13	1.29	1.52	1.79	1.86	1.91	2.12	2.74	3.53
21	1.78	1.28	0.85	1.00	0.77	0.90	0.78	0.83	1.34	1.45	1.22
22	2.50	2.54	2.42	2.49	2.48	2.48	2.38	2.44	2.81	3.04	3.60
23	3.00	3.23	3.21	3.27	3.25	3.29	3.19	3.39	3.29	3.87	4.68
24	2.01	1.88	1.69	1.74	1.78	1.75	1.79	1.98	1.70	2.09	2.70
25	2.66	2.61	2.71	2.65	2.87	2.68	2.95	3.19	3.11	2.94	2.75
26	2.37	1.75	1.59	2.04	1.90	1.86	1.89	1.80	2.84	3.75	4.20
27A	2.67	2.12	2.02	2.04	2.02	2.15	2.10	1.93	2.57	3.30	3.89
Average	**1.95**	**1.92**	**1.86**	**1.95**	**1.92**	**1.95**	**2.00**	**2.07**	**2.44**	**2.97**	**3.38**
r.m.s.	**0.72**	**0.77**	**0.83**	**0.82**	**0.82**	**0.80**	**0.78**	**0.82**	**0.84**	**1.12**	**1.31**

Table A3 - Sextupole contribution divided by ideal quadrupole field
on a circle of 30 mm radius around the magnet axis (units of 10^{-4})

Serial#	18.10A	36.19A	54.28A	64.65A	72.36A	90.45A	108.54A	126.63A	145.94A	162.75A	180.80A
0	1.61	1.65	1.76	1.99	1.92	1.72	1.88	2.08	1.74	1.46	1.75
1	2.79	2.49	2.59	2.75	2.63	2.74	2.57	2.77	4.13	5.26	5.46
2	1.32	0.95	1.01	1.21	1.13	1.46	1.39	1.49	1.48	1.75	2.03
3	0.87	0.99	0.99	0.88	1.23	1.05	1.24	1.43	2.58	3.28	3.70
4	2.18	1.98	2.06	2.26	2.28	2.12	2.33	2.57	3.29	3.91	4.32
5	0.60	1.10	0.98		1.01	1.31	1.14	1.40	1.43	2.27	3.08
6	1.61	2.36	2.20	2.36	2.43	2.64	2.75	3.03	3.12	3.86	4.41
7	0.89	0.46	0.84	0.84	0.84	0.86	0.99	1.04	1.51	1.90	2.49
8	3.53	3.85	4.07	4.11	4.20	4.14	4.11	4.15	3.51	3.08	2.67
9	0.92	1.02	1.20	1.12	1.33	1.15	1.43	1.07	1.58	2.38	2.70
10	1.75	2.09	2.07	1.97	1.93	1.98	2.02	1.96	2.77	3.51	3.70
11	1.02	1.39	0.97	1.07	1.19	0.81	1.13	1.37	2.80	4.31	5.56
12	1.01	1.03	1.04	0.89	0.89	0.89	1.03	1.13	0.94	1.02	1.35
13	1.58	2.04	2.03	2.02	1.93	1.98	1.92	1.75	1.35	1.93	2.22
14	1.75	1.99	2.19	2.13	2.00	2.07	2.26	2.42	3.22	4.90	6.20
15	2.02	2.08	1.48	1.75	1.46	1.49	1.48	1.63	3.24	4.43	4.52
16	0.32	0.92	0.87	0.94	0.98	1.14	0.96	0.97	1.42	1.15	0.45
17	1.04	0.85	0.48	0.62	0.99	0.82	1.08	1.33	1.04	0.54	0.79
18	0.65	0.22	0.14	0.17	0.13	0.36	0.47	0.59	0.96	1.97	2.71
19	1.88	1.35	1.59	1.60	1.31	1.24	1.86	1.45	1.93	2.97	3.64
20	1.12	0.67	0.94	1.15	1.41	1.71	1.78	1.83	2.04	2.68	3.45
21	1.63	1.18	0.73	0.90	0.63	0.79	0.61	0.56	1.24	1.35	1.08
22	0.56	0.48	0.35	0.70	0.53	0.52	0.53	0.60	1.18	2.28	3.04
23	1.99	2.43	2.43	2.49	2.41	2.49	2.44	2.64	2.53	3.39	4.28
24	1.18	0.95	0.73	0.77	0.81	0.80	0.82	1.03	0.65	1.46	2.39
25	2.06	2.14	2.39	2.23	2.60	2.34	2.67	2.93	2.84	2.64	2.45
26	2.21	1.61	1.43	1.88	1.76	1.73	1.75	1.65	2.75	3.68	4.13
27A	2.38	1.79	1.68	1.70	1.71	1.84	1.78	1.57	2.33	3.12	3.75
Average	**1.52**	**1.50**	**1.47**	**1.58**	**1.56**	**1.58**	**1.66**	**1.73**	**2.13**	**2.73**	**3.15**
r.m.s.	**0.73**	**0.79**	**0.85**	**0.85**	**0.84**	**0.82**	**0.81**	**0.84**	**0.93**	**1.23**	**1.42**

Table A4 - Octupole contribution divided by ideal quadrupole field on a circle of 30 mm radius around the magnet axis (units of 10^{-4})

Serial#	18.10A	36.19A	54.28A	64.65A	72.36A	90.45A	108.54A	126.63A	145.94A	162.75A	180.80A
0	2.51	2.54	2.50	2.33	2.45	2.39	2.35	2.32	2.17	2.15	2.28
1	0.55	0.61	0.58	0.53	0.56	0.53	0.54	0.58	0.53	0.54	0.59
2	0.67	0.60	0.40	0.52	0.64	0.54	0.55	0.57	0.59	0.54	0.51
3	0.98	1.08	1.12	1.08	1.07	1.11	1.04	1.01	1.02	1.04	1.04
4	0.61	0.38	0.43	0.38	0.45	0.38	0.34	0.43	0.37	0.45	0.39
5	0.63	0.52	0.58		0.54	0.56	0.52	0.60	0.58	0.62	0.57
6	0.72	0.38	0.45	0.37	0.38	0.39	0.41	0.40	0.32	0.39	0.32
7	1.36	1.37	1.13	1.19	1.22	1.22	1.12	1.18	1.15	1.25	1.19
8	1.85	1.92	1.82	1.84	1.78	1.85	1.83	1.87	1.82	1.84	1.81
9	0.42	0.35	0.33	0.38	0.40	0.33	0.51	0.39	0.40	0.40	0.41
10	0.53	0.57	0.59	0.54	0.54	0.55	0.57	0.48	0.51	0.54	0.46
11	0.41	0.57	0.70	0.58	0.50	0.69	0.65	0.58	0.62	0.56	0.60
12	1.17	1.14	1.15	1.15	0.99	1.19	1.11	1.09	1.13	1.05	1.24
13	1.83	1.90	1.87	1.78	1.82	1.77	1.85	1.83	1.88	1.86	1.88
14	0.80	0.86	0.74	0.93	0.85	0.83	0.78	0.75	0.74	0.76	0.78
15	1.38	1.21	0.99	1.03	0.99	1.02	1.02	0.95	0.97	0.99	0.93
16	0.88	0.91	0.83	0.81	0.77	0.65	0.80	0.85	0.81	0.69	0.97
17	0.32	0.33	0.26	0.32	0.11	0.34	0.31	0.20	0.29	0.32	0.12
18	0.70	0.63	0.60	0.63	0.65	0.58	0.61	0.57	0.62	0.49	0.50
19	1.09	0.99	0.74	0.85	0.69	0.77	0.60	0.78	0.53	0.73	0.55
20	0.30	0.56	0.39	0.34	0.22	0.25	0.24	0.34	0.37	0.21	0.30
21	0.65	0.29	0.03	0.12	0.09	0.06	0.12	0.13	0.13	0.17	0.07
22	2.45	2.49	2.39	2.41	2.42	2.43	2.32	2.37	2.54	2.26	2.32
23	2.68	2.59	2.58	2.59	2.64	2.63	2.52	2.60	2.59	2.48	2.55
24	1.75	1.70	1.56	1.62	1.65	1.62	1.65	1.80	1.52	1.55	1.47
25	1.71	1.75	1.64	1.82	1.60	1.65	1.64	1.65	1.63	1.65	1.58
26	0.93	0.71	0.74	0.85	0.75	0.75	0.79	0.78	0.72	0.78	0.68
27A	1.50	1.37	1.34	1.38	1.32	1.38	1.37	1.36	1.39	1.37	1.33
Average	**1.12**	**1.08**	**1.02**	**1.05**	**1.00**	**1.01**	**1.01**	**1.02**	**1.00**	**0.99**	**0.98**
r.m.s.	**0.68**	**0.71**	**0.70**	**0.70**	**0.71**	**0.71**	**0.68**	**0.70**	**0.69**	**0.66**	**0.69**

Table A5 - Twelve-pole contribution divided by ideal quadrupole field on a circle of 30 mm radius around the magnet axis (units of 10^{-4})

Serial#	**18.10A**	**36.19A**	**54.28A**	**64.65A**	**72.36A**	**90.45A**	**108.54A**	**126.63A**	**145.94A**	**162.75A**	**180.80A**
0	0.471	0.456	0.455	0.449	0.428	0.455	0.440	0.503	0.537	0.587	0.645
1	0.512	0.453	0.425	0.463	0.421	0.475	0.478	0.509	0.549	0.600	0.636
2	0.560	0.466	0.462	0.487	0.483	0.480	0.501	0.482	0.544	0.613	0.679
3	0.499	0.453	0.465	0.479	0.449	0.490	0.474	0.514	0.557	0.598	0.653
4	0.523	0.505	0.487	0.443	0.487	0.525	0.490	0.542	0.566	0.610	0.660
5	0.475	0.467	0.431		0.479	0.460	0.499	0.483	0.535	0.585	0.642
6	0.471	0.437	0.453	0.463	0.453	0.477	0.471	0.509	0.569	0.562	0.639
7	0.494	0.414	0.455	0.448	0.434	0.460	0.510	0.539	0.559	0.556	0.613
8	0.518	0.462	0.468	0.443	0.461	0.501	0.505	0.521	0.572	0.601	0.636
9	0.500	0.471	0.408	0.441	0.438	0.435	0.451	0.475	0.525	0.566	0.649
10	0.537	0.449	0.429	0.445	0.481	0.469	0.451	0.555	0.553	0.619	0.691
11	0.543	0.508	0.527	0.511	0.542	0.518	0.546	0.563	0.594	0.647	0.709
12	0.549	0.472	0.489	0.516	0.514	0.520	0.525	0.584	0.572	0.634	0.691
13	0.466	0.423	0.416	0.415	0.457	0.434	0.453	0.496	0.512	0.575	0.602
14	0.552	0.439	0.467	0.429	0.398	0.470	0.493	0.482	0.508	0.602	0.607
15	0.552	0.434	0.459	0.438	0.447	0.444	0.458	0.493	0.528	0.574	0.617
16	0.376	0.489	0.468	0.444	0.446	0.463	0.407	0.491	0.606	0.655	0.701
17	0.486	0.512	0.432	0.477	0.478	0.533	0.517	0.484	0.571	0.595	0.726
18	0.510	0.470	0.512	0.433	0.468	0.491	0.496	0.553	0.530	0.643	0.654
19	0.582	0.481	0.514	0.423	0.363	0.460	0.486	0.455	0.518	0.549	0.612
20	0.553	0.510	0.554	0.545	0.548	0.537	0.568	0.514	0.573	0.628	0.821
21	0.571	0.468	0.466	0.444	0.458	0.463	0.473	0.630	0.563	0.617	0.662
22	0.528	0.461	0.493	0.487	0.475	0.480	0.611	0.554	0.716	0.650	0.721
23	0.457	0.410	0.386	0.444	0.418	0.388	0.468	0.477	0.503	0.560	0.573
24	0.633	0.446	0.454	0.416	0.501	0.507	0.558	0.519	0.570	0.596	0.646
25	0.633	0.502	0.450	0.504	0.487	0.553	0.471	0.526	0.581	0.634	0.672
26	0.550	0.490	0.461	0.475	0.494	0.484	0.480	0.484	0.604	0.582	0.667
27A	0.383	0.339	0.322	0.349	0.353	0.339	0.349	0.429	0.430	0.484	0.570
Average	**0.517**	**0.460**	**0.457**	**0.456**	**0.459**	**0.475**	**0.487**	**0.513**	**0.555**	**0.597**	**0.657**
r.m.s.	**0.059**	**0.037**	**0.045**	**0.039**	**0.045**	**0.045**	**0.050**	**0.042**	**0.048**	**0.037**	**0.051**

Table A6 - 20-pole contribution divided by ideal quadrupole field
on a circle of 30 mm radius around the magnet axis (units of 10^{-4})

Serial#	**18.10A**	**36.19A**	**54.28A**	**64.65A**	**72.36A**	**90.45A**	**108.54A**	**126.63A**	**145.94A**	**162.75A**	**180.80A**
0	0.303	0.303	0.301	0.312	0.306	0.307	0.309	0.315	0.312	0.318	0.308
1	0.300	0.311	0.315	0.308	0.311	0.307	0.315	0.311	0.319	0.309	0.315
2	0.316	0.306	0.311	0.310	0.310	0.319	0.313	0.316	0.317	0.309	0.310
3	0.318	0.315	0.308	0.315	0.312	0.312	0.310	0.317	0.314	0.318	0.317
4	0.323	0.311	0.318	0.315	0.306	0.313	0.310	0.313	0.317	0.312	0.320
5	0.311	0.309	0.311		0.306	0.316	0.311	0.317	0.313	0.310	0.315
6	0.308	0.303	0.303	0.307	0.315	0.309	0.315	0.316	0.313	0.310	0.312
7	0.315	0.313	0.310	0.316	0.316	0.312	0.323	0.316	0.316	0.319	0.312
8	0.314	0.306	0.307	0.310	0.310	0.318	0.315	0.316	0.315	0.308	0.312
9	0.323	0.312	0.305	0.312	0.317	0.312	0.301	0.306	0.314	0.319	0.326
10	0.317	0.301	0.312	0.311	0.312	0.303	0.305	0.308	0.305	0.319	0.315
11	0.310	0.315	0.311	0.309	0.312	0.314	0.308	0.312	0.310	0.306	0.315
12	0.308	0.307	0.313	0.313	0.310	0.304	0.326	0.309	0.316	0.317	0.307
13	0.322	0.313	0.313	0.315	0.312	0.315	0.317	0.316	0.307	0.317	0.314
14	0.311	0.315	0.310	0.316	0.314	0.307	0.306	0.309	0.320	0.318	0.323
15	0.313	0.308	0.304	0.318	0.318	0.311	0.309	0.320	0.314	0.313	0.316
16	0.322	0.327	0.306	0.310	0.316	0.310	0.297	0.325	0.321	0.304	0.341
17	0.308	0.318	0.313	0.317	0.317	0.306	0.312	0.324	0.315	0.307	0.310
18	0.316	0.318	0.309	0.313	0.316	0.318	0.316	0.322	0.310	0.314	0.300
19	0.307	0.335	0.296	0.335	0.328	0.315	0.296	0.305	0.320	0.300	0.327
20	0.304	0.308	0.309	0.334	0.324	0.318	0.304	0.326	0.322	0.333	0.313
21	0.319	0.315	0.315	0.321	0.303	0.313	0.314	0.295	0.306	0.309	0.308
22	0.296	0.323	0.313	0.307	0.321	0.315	0.297	0.307	0.300	0.312	0.300
23	0.310	0.314	0.307	0.297	0.308	0.312	0.308	0.310	0.312	0.317	0.320
24	0.324	0.336	0.312	0.318	0.314	0.308	0.323	0.359	0.317	0.309	0.304
25	0.304	0.310	0.307	0.329	0.305	0.323	0.308	0.284	0.323	0.306	0.323
26	0.309	0.312	0.312	0.319	0.314	0.316	0.316	0.309	0.315	0.313	0.315
27A	0.311	0.315	0.309	0.313	0.315	0.318	0.319	0.319	0.327	0.315	0.320
Average	**0.312**	**0.314**	**0.309**	**0.315**	**0.313**	**0.312**	**0.311**	**0.314**	**0.314**	**0.313**	**0.315**
r.m.s.	**0.007**	**0.008**	**0.005**	**0.008**	**0.006**	**0.005**	**0.008**	**0.012**	**0.006**	**0.006**	**0.008**