DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **L-34**

THE WIGGLER TRANSFER MATRIX

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1. Description of the method

The transfer matrix of the wiggler can be calculated, starting from the measurement of the vertical field component on the symmetry plane of the magnet, by calculating the trajectory of a test particle as a function of its initial conditions.

The equations of motion for a positron in the wiggler field are given by the Lorentz force:

$$\frac{\partial^2 x}{\partial t^2} = a\left(\frac{\partial y}{\partial t}B_z - \frac{\partial z}{\partial t}B_y\right)$$

$$\frac{\partial^2 y}{\partial t^2} = a\left(\frac{\partial z}{\partial t}B_x - \frac{\partial x}{\partial t}B_z\right)$$

$$\frac{\partial^2 z}{\partial t^2} = a\left(\frac{\partial x}{\partial t}B_y - \frac{\partial y}{\partial t}B_x\right)$$

with

$$a = \frac{e}{m_o\gamma} = \frac{8.98753x10^{10}}{E(MeV)};e = 1.6021\times10^{-19}C;m_0 = 9.1091\times10^{-31}Kg$$

and the condition:

$$\left(\frac{\partial x}{\partial t}\right)^2 + \left(\frac{\partial y}{\partial t}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2 = v^2 = c^2\sqrt{1-\frac{1}{\gamma^2}} \approx c^2\left(1-\frac{1}{2\gamma^2}\right)$$

Due to the symmetry of the structure with respect to the horizontal symmetry plane of the magnet, the horizontal and longitudinal components of the field (B_x and B_z) vanish on this plane:

$$\gamma = \frac{E}{m_o c^2} = \frac{E(MeV)}{0.511006}$$

$$B_x(x,0,z) = B_z(x,0,z) = 0$$

Inside the transverse displacement of the trajectory in the wiggler with respect to the magnet axis (±12 mm at 1.71T and 510 MeV) the transverse behaviour of the vertical field component is very well fitted with a second order polynomial:

$$B_y(x,0,z) = b_0(z) + b_1(z)x + b_2(z)x^2$$

while the longitudinal dependence of the three coefficients of the transverse expansion asks for a higher order polynomial to achieve a satisfactory agreement. It has been found that a fourth order polynomial allows to fit six consecutive points (corresponding to measurements taken at an interval of 8.35 mm) with a negligible deviation. We have therefore:

$$\begin{aligned} b_0(z) &= b_{00i} + b_{01i}z + b_{02i}z^2 + b_{03i}z^3 + b_{04i}z^4 \\ b_1(z) &= b_{10i} + b_{11i}z + b_{12i}z^2 + b_{13i}z^3 + b_{14i}z^4 \\ b_2(z) &= b_{20i} + b_{21i}z + b_{22i}z^2 + b_{23i}z^3 + b_{24i}z^4 \end{aligned}$$

where the index i indicates that the corresponding coefficients are the those belonging to the fourth order polynomial fitting three measured points on the field map before the longitudinal position z and three after it.

The horizontal field component at a vertical position y outside the horizontal symmetry plane can be estimated, to first order, from the curl theorem:

$$B_x(x,y,z) \approx \frac{\partial B_x}{\partial y}y = \frac{\partial B_y}{\partial x}y = \left[b_1(z) + 2b_2(z)x\right]y$$

while the longitudinal component is given by:

$$B_z(x,y,z) \approx \frac{\partial B_z}{\partial y}y = \frac{\partial B_y}{\partial z}y = \left[\frac{\partial b_0(z)}{\partial z} + \frac{\partial b_1(z)}{\partial z}x + \frac{\partial b_2(z)}{\partial z}x^2\right]y$$

$$\frac{\partial b_0(z)}{\partial z} = b_{01i} + 2b_{02i}z + 3b_{03i}z^2 + 4b_{04i}z^3$$

$$\frac{\partial b_1(z)}{\partial z} = b_{11i} + 2b_{12i}z + 3b_{13i}z^2 + 4b_{14i}z^3$$

$$\frac{\partial b_2(z)}{\partial z} = b_{21i} + 2b_{22i}z + 3b_{23i}z^2 + 4b_{24i}z^3$$

Finally, the vertical field component outside the horizontal symmetry plane can be calculated from the divergence theorem:

$$B_y(x,y,z) \approx B_y(x,0,z) + \frac{\partial B_y}{\partial y} y = B_y(x,0,z) - y\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z}\right) =$$

$$= B(x,0,z) - y^2\left[2b_2(z) + \frac{\partial^2 b_0(z)}{\partial z^2} + \frac{\partial^2 b_1(z)}{\partial z^2} x + \frac{\partial^2 b_2(z)}{\partial z^2} x^2\right]$$

$$\frac{\partial^2 b_0(z)}{\partial z^2} = 2b_{02i} + 6b_{03i} z + 12 b_{04i} z^2$$

$$\frac{\partial^2 b_1(z)}{\partial z^2} = 2b_{12i} + 6b_{13i} z + 12 b_{14i} z^2$$

$$\frac{\partial^2 b_2(z)}{\partial z^2} = 2b_{22i} + 6b_{23i} z + 12 b_{24i} z^2$$

2. Results

A program has been written in Fortran which solves the equations of motion with the field approximations described in the previous section. As a first test, it has been applied to a simplified wiggler map where the measurements performed in August and September 2003 [1] on half central pole have been reflected and replicated with the suitable sign over the five full wiggler poles. The fields measured on the terminal pole have been added on each side after multiplying all values by a coefficient in order to make the field integral on the whole wiggler axis vanish. Only the measurements taken at intervals of 1 cm between ±3 cm from the wiggler axis have been taken into account.

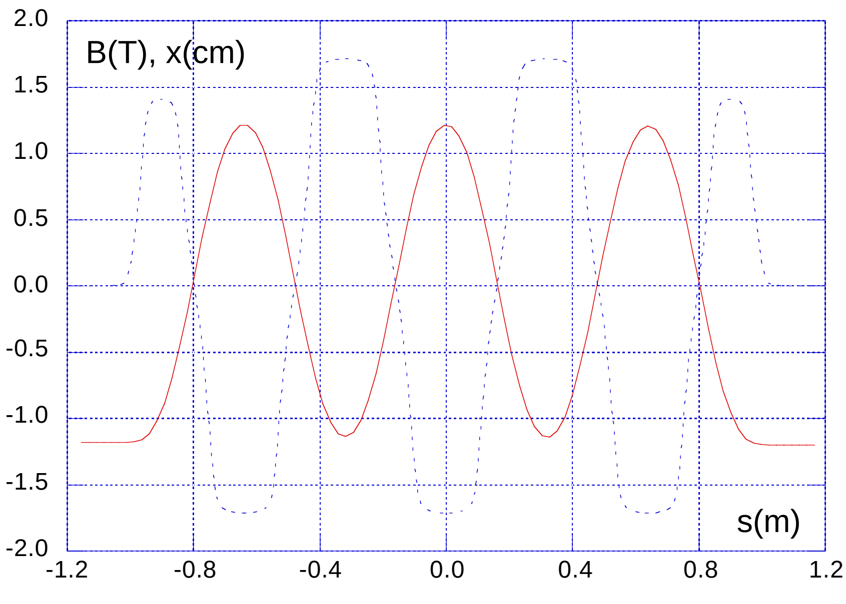

Fig. 1 - b_0 (T), dotted line - Beam trajectory in the horizontal plane, full line (cm)

Figure 1 shows the behaviour of the first coefficient b_0 together with the resulting trajectory in the horizontal plane starting at -1.18 cm from the wiggler axis. Figures 2 and 3 show the longitudinal behaviour of $b_1(z)$ and $b_2(z)$ respectively. Figure 4 shows a trajectory in the vertical plane starting at 1 mm from the axis.

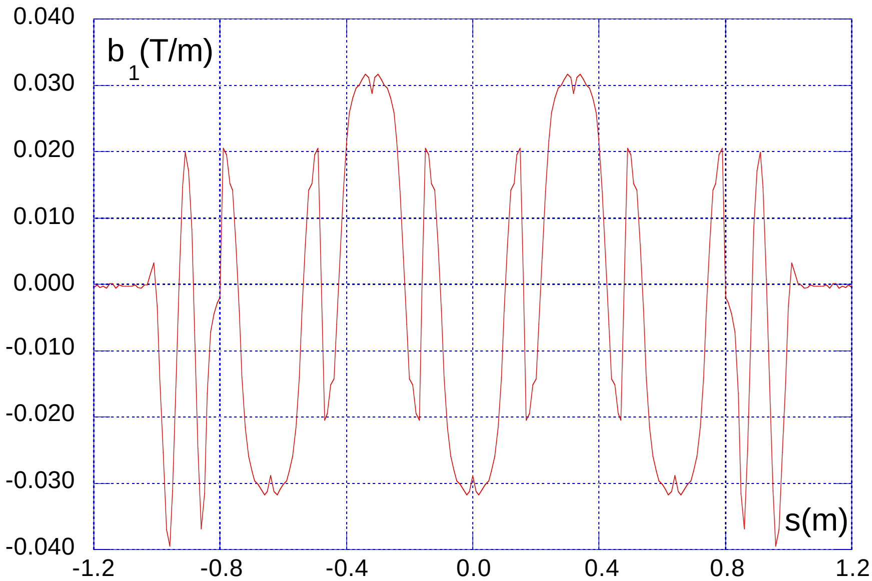

Fig. 2 - b_1 (T/m)

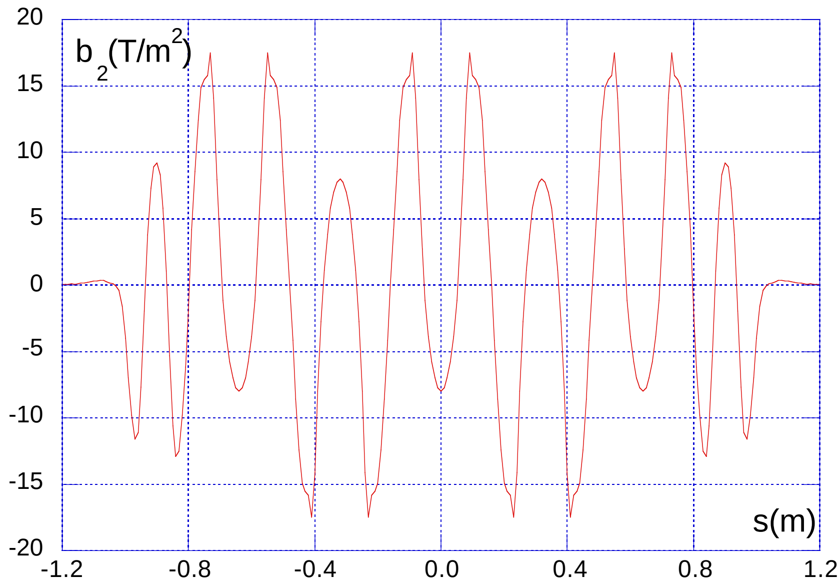

Fig. 3 - b_2 (T/m^2)

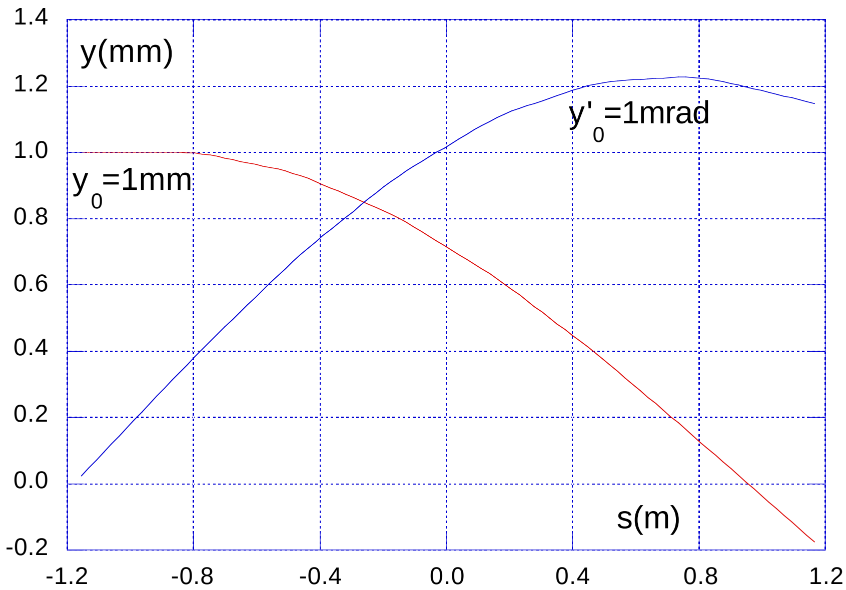

Fig. 4 Beam trajectory in the vertical plane starting at 1 mm from the horizontal symmetry plane and with 1 mrad with respect to the symmetry plane

The program calculates also the length of the trajectory: in the nominal case of Figure 1 the lengthening of the beam path with respect to a straight line on the axis is 6.58 mm. The 5x5 transfer matrix of the wiggler is calculated by tracing the trajectories of the particles with small initial displacements, angles and energy deviations with respect to the nominal ones and comes out to be on a total wiggler length of 2.36 m:

1.05614	2.43128	–0.00021	–0.00026	–0.00049
0.04749	1.05611	–0.00016	–0.00021	–0.00043
0	0	1	0	0
0	0	0	–0.18761	1.15611
0	0	0	–0.83450	–0.18765

References

[1] - DAFNE Technical Note on the wiggler measurements. To be published.