DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **L-12**

BEAM LIFETIME IN DAΦNE

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1. Introduction

The choice of the vacuum chamber aperture for DAΦNE has been made taking into account the technical constraints as well as all the effects limiting the beam lifetime; a detailed discussion is given in Ref. [1]. The beam lifetime is mainly determined by the single Touschek scattering and by the gas scattering. Among the two effects the Touschek is the dominant one, due to the relatively low energy, on the assumption that the vacuum is in the 10^{-9} Torr range.

The Touschek beam lifetime has been calculated by the computer code LEDA [2]. To get a more precise estimate the code has been modified in order to take into account the actual shape of the vacuum chamber and the limit due to the dynamic aperture.

First of all an estimation of the beam lifetime requirements for DAΦNE is presented in **Section 2.** In **Section 3** a description of the algorithms used to evaluate the Touschek beam lifetime is given. In **Sections 4** and **5** the results of the calculations are reported together with a discussion of the physical and dynamic aperture required to obtain a satisfactory beam lifetime, for single beam and two beams operation.

2. Required beam lifetime

The beam lifetime required in a collider depends on the ratio of the average luminosity respect to the peak one in a machine run. An exact evaluation of the behaviour of luminosity versus time requires too many hypotheses on the beam-beam interaction, beam dimensions and gas pressure in the vacuum chamber, but to have an estimate of the required beam lifetime we can make some simplifying assumptions.

We assume that, by reducing the coupling factor or the emittance proportionally to the current, the value of the linear tune shift parameter can be kept constant and therefore the luminosity decreases linearly with the current during the run. For the time dependence of the current we use an exponential behaviour with a time constant calculated for the initial current. In this approximation we neglect the dependence of the gas pressure on the current because it affects only the gas scattering beam lifetime which is a minor contribution to the total beam lifetime. We have neglected also the weak bunch length variation with the current since, in the turbulent regime, it is proportional to the cubic root of the current.

Under these assumptions, a first estimate of the ratio between the average and peak luminosity as a function of the injection time t_{inj}, the run duration t_{run} and the beam lifetime τ is given by a very simple equation:

$$\frac{\langle L \rangle}{L_{peak}} = \frac{1 - e^{-\frac{t_{run}}{\tau}}}{\frac{t_{inj}}{\tau} + \frac{t_{run}}{\tau}}$$

The behaviour of <L>/L_{peak} as a function of t_{run}/τ, with t_{inj}/τ as a parameter, is shown in **Fig. 1**. As t_{run} can be arbitrarily chosen in order to get the maximum of <L>/L_{peak} for a given value of t_{inj}/τ, the only relevant parameter is the ratio between injection time and beam lifetime t_{inj}/τ and not the beam lifetime itself.

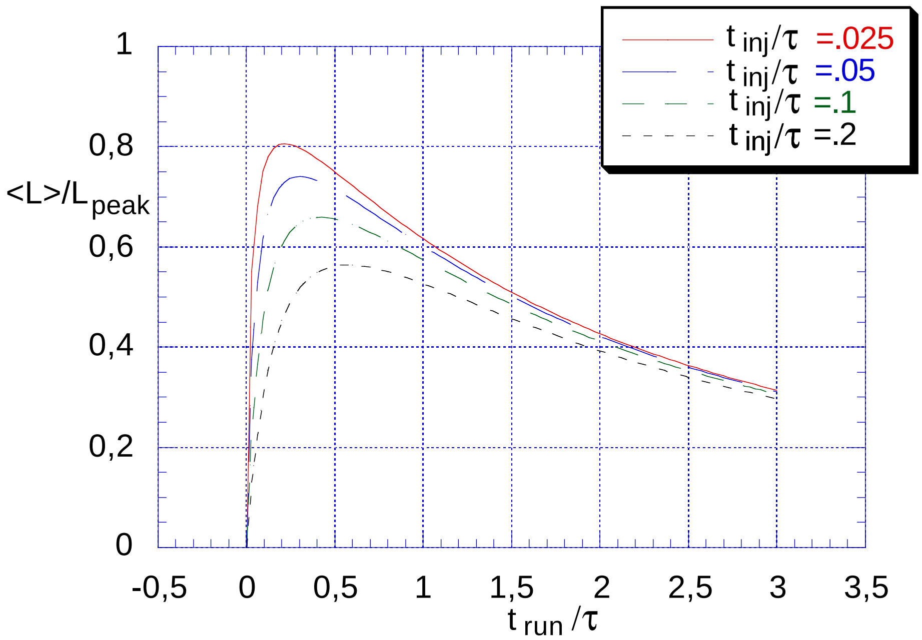


Fig. 1 - <L>/L_{peak} versus t_{run}/τ for different values of t_{inj}/τ.

To maximize the ratio between the average and peak luminosity, which is the relevant parameter for the performance of a collider, one can increase the beam lifetime or decrease, by the same factor, the injection time.

In **Fig. 2** the maximum of the ratio <L>/L_{peak} and the corresponding run duration t_{run}/τ are shown as a function of t_{inj}/τ.

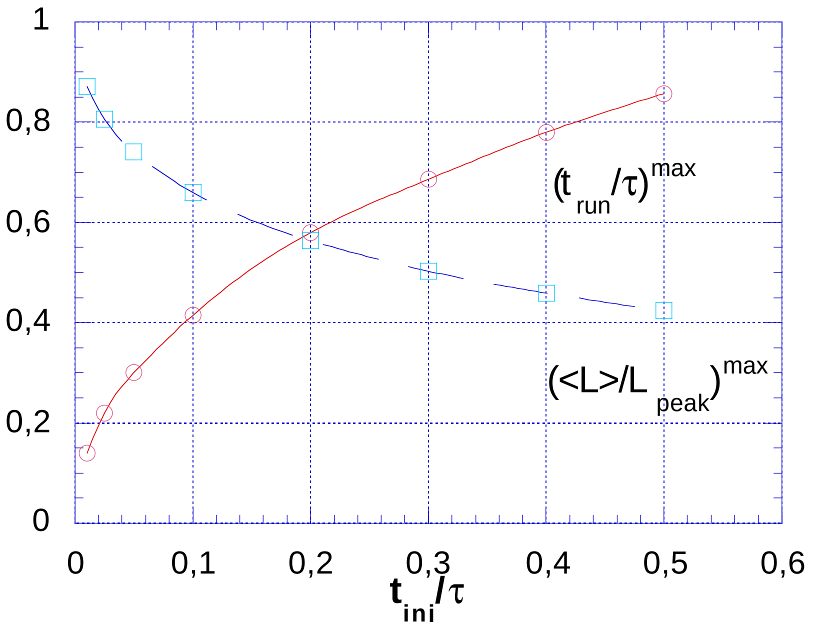


Fig. 2 - The maximum of the ratio $<L>/L_{peak}$ and the corresponding run duration t_{run}/τ as a function of t_{inj}/τ.

The design objectives for DAΦNE are an injection time of the order of 10 minutes for the maximum current in both rings and a 3÷4 hours beam lifetime. This corresponds to an average luminosity as high as 75% of the peak value with a run duration of the order of one hour (see the curve for t_{inj}/τ=.05 in Fig. 1).

It is quite demanding to reach these values of injection time and beam lifetime, and, for practical reasons, to have such a short run duration. Anyway with these parameters a longer run duration gives only a small reduction in the ratio $<L>/L_{peak}$.

In **Table I** the values of the ratio $<L>/L_{peak}$ are summarized, for one or three hours run duration, with the design value of the ratio t_{inj}/τ and in the pessimistic assumption of a twice larger t_{inj}/τ.

Table I - Ratio between average and peak luminosity

	t_{inj}/τ **= .05** (t_{inj} = 10 min τ = 200 min)	t_{inj}/τ **= .1** (t_{inj} = 15 min τ = 150 min)
t_{run}=1h	$<L>/L_{peak}$=.74	$<L>/L_{peak}$=.66
t_{run}=3h	$<L>/L_{peak}$=.62	$<L>/L_{peak}$=.54

3. Touschek scattering lifetime calculation

The beam lifetime due to single Touschek scattering is proportional to the third power of the energy, and therefore it is the main limitation for low energy storage rings like DAΦNE.

The Touschek half-lifetime is calculated according to the formula given by H. Brook[3]:

$$\frac{1}{\tau} = \frac{\sqrt{\pi}\, r_o^2 c N}{\gamma^3 \sigma'_x \varepsilon^2 (4\pi)^{\frac{3}{2}} \sigma_l \sigma_x \sigma_y} C(u_{min})$$

where:

r_o = classical electron radius
c = velocity of light
γ = electron energy in units of its rest mass
N = number of electrons per bunch
σ'_x = angular divergence of the beam
$(4\pi)^{3/2}\sigma_l\sigma_x\sigma_y$ = beam volume

and

$$C(u_{min}) = \int_{u_{min}}^{\infty} \frac{1}{u^2}\left[u - u_{min} - \frac{1}{2} ln\left(\frac{u}{u_{min}}\right)\right] e^{-u}\, du$$

with

$$u_{min} = \left(\frac{\varepsilon}{\gamma\sigma'_x}\right)^2$$

$$\sigma'_x = \left[\varepsilon_x/\beta_x + \sigma_p^2 (D'_x + D_x\alpha_x/\beta_x)^2\right]^{1/2}$$

and ε = limiting acceptance for the relative momentum deviation of a particle which undergoes a large angle Touschek scattering. It is the minimum between the RF acceptance and the momentum acceptance due to the transverse aperture, either physical or dynamic.

3.1 Physical aperture limitation for the momentum acceptance

At each azimuth s_i along the ring the following quantity is calculated (only for the horizontal plane because the vertical dispersion is zero):

$$H(s_i) = \gamma_i D_i^2 + 2\alpha_i D_i D'^2_i + \beta_i D'^2_i .$$

The maximum horizontal displacement in a position s_j for a particle which has got a relative momentum deviation change ε_i in s_i is:

$$x_j = \varepsilon_i \left[\sqrt{H_i \beta_j} + |D_j| \right]$$

The limiting value for ε_i is obtained by equating x_j to the physical half-aperture in that position A_x^j and taking the minimum all over the ring:

$$\varepsilon_i = \min_j \left\{ \frac{A_x^j}{\sqrt{H_i \beta_j} + |D_j|} \right\}$$

3.2 Dynamic aperture limitation for the momentum acceptance

The check on the dynamic aperture acceptance is done only at one point s_o (where the dynamic aperture is calculated) since we assume that in the rest of the ring it scales with the square root of the unperturbed optical functions.

To calculate the momentum acceptance due to the dynamic aperture limitation the maximum stable oscillation amplitude in the horizontal plane as a function of the relative momentum deviation $A_{DA}(\varepsilon)$ is needed.

The limiting value of the relative momentum deviation ε_i is obtained for each point s_i by solving the following equation iteratively:

$$\varepsilon = \frac{A_{DA}(\varepsilon)}{\sqrt{H_i \beta_0} + |D_0|}$$

3.3 Modifications to the LEDA code

The total beam lifetime is calculated by the LEDA code keeping into account quantum lifetime, Touschek scattering, elastic and inelastic gas scattering.

The minimum between the RF acceptance and the two values obtained above for the vacuum chamber and dynamic aperture limitation is used to evaluate the Touschek half-life-time at the azimuth s_i. The final result is obtained as an average all over the ring circumference:

$$\frac{1}{\tau} = \frac{\int_C \frac{1}{\tau(s)} ds}{\int_C ds}$$

The physical aperture is given in the input data for each element in the eighth field; if it is not specified a constant value x_0 is assumed.

The value of the horizontal limit of the dynamic aperture is given at a point s_0 in the ring together with the β_x and D_x values at that position. As we do not have an equation for A_{DA} as a function of energy, but a discrete number of values, a step function is assumed (see the example in **Fig. 3**).

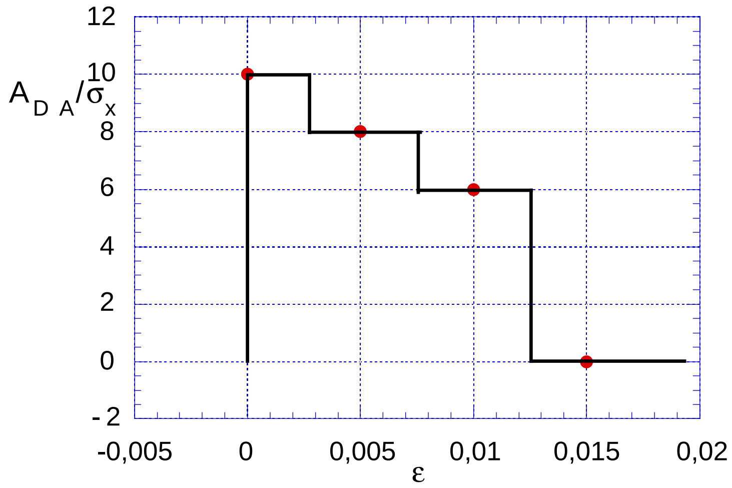


Fig. 3 - Schematization of A_{DA}, in units of σ_x, as a function of ε adopted to calculate the Touschek beam lifetime. The dots correspond to the computed dynamic apertures.

Two vectors, one with the energy values and one with the corresponding values of the horizontal limit of the dynamic aperture at that energy, are given as input data.

The bunch length and the energy spread with the DAΦNE design parameters are in the anomalous lengthening regime and therefore the beam size and divergence are enlarged in the region where the dispersion is not zero taking into account the increase in the energy spread.

An example of the LEDA input and output file is shown in **Appendix A**, the differences with respect to the previous version are underlined.

4. Results

All the results are for the DAY-ONE lattice, but we do not expect significant differences for the other lattices since the optical functions are the same in the arcs and very similar in the interaction regions (same β_{max}, zero dispersion).

The parameters used to calculate the beam lifetime are shown in **Table II**. These are the design values corresponding to the peak luminosity [4], in particular maximum bunch peak current and minimum coupling and therefore this is the worst expected beam lifetime.

Table II - DAΦNE design parameters

Energy (MeV)	510.
RF Voltage V_{RF} (KV)	190.
RF acceptance ε_{RF}	1.8%
N of particles /bunch	$8.9\ 10^{10}$
Average current /bunch (mA)	43.75
Estimated impedance Z/n (Ω)	.5
Bunch length σ_l (cm)	3.
Relative energy spread	$1.5\ 10^{-3}$
Momentum compaction	.006
Natural emittance (m·rad)	10^{-6}
Coupling factor κ	.01
Vertical half-aperture @β_{MAX} A_y (cm)	4.6

All the contributions to the beam lifetime and its total value, calculated with the above parameters and the effective physical aperture, are listed in **Table III** for a residual gas pressure $P=10^{-9}$ Torr (biatomic gas, Z=8). The horizontal aperture along the ring is shown in **Fig. 4**.

Table III - Contributions to the DAΦNE beam lifetime

Quantum life energy osc. (hours)	$3.2\ 10^{22}$
Quantum life radial osc. (hours)	$7.0\ 10^{13}$
Gas bremsstrahlung nuclei (min)	$2.35\ 10^{3}$
Gas scattering nuclei (min)	$1.43\ 10^{3}$
Gas bremsstrahlung e^- (min)	$1.45\ 10^{4}$
Gas scattering e^- (min)	$3.23\ 10^{4}$
Touschek scattering (min)	342.
Total lifetime (min)	241.
Two beams lifetime @ $L = 6.0\ 10^{32}\ cm^{-2}s^{-1}$	
Beam-beam bremsstrahlung (min)	1734.
Total lifetime (min)	212.

The Touschek scattering is the dominant contribution to the total beam lifetime and is determined by the RF acceptance and the vacuum chamber aperture. The Touschek lifetime and total one, as a function of the RF voltage and the corresponding RF acceptance, are shown in **Table IV** and **Fig. 5**. The design RF voltage is in the range between 130 and 250 KV and therefore very near to the maximum beam lifetime achievable with the physical aperture.

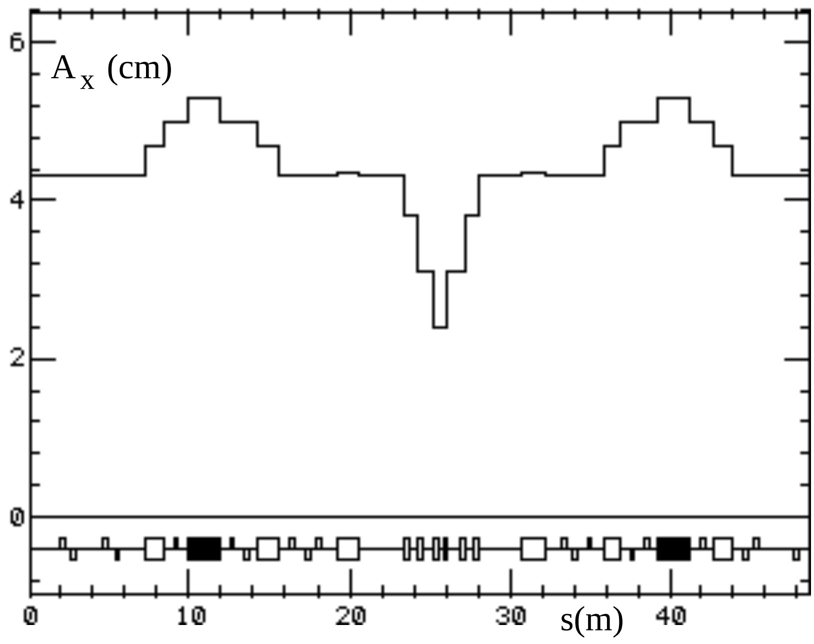


Fig. 4 - Horizontal aperture for half ring between two symmetry points.

Table IV

V_{RF} (KV)	ε_{RF}	τ_{Tou} (min)	τ_{tot} (min)
95.	$1.19\ 10^{-2}$	266.	198.
190.	$1.76\ 10^{-2}$	342.	241.
297.	$2.20\ 10^{-2}$	365.	255.
380.	$2.54\ 10^{-2}$	365.	256.

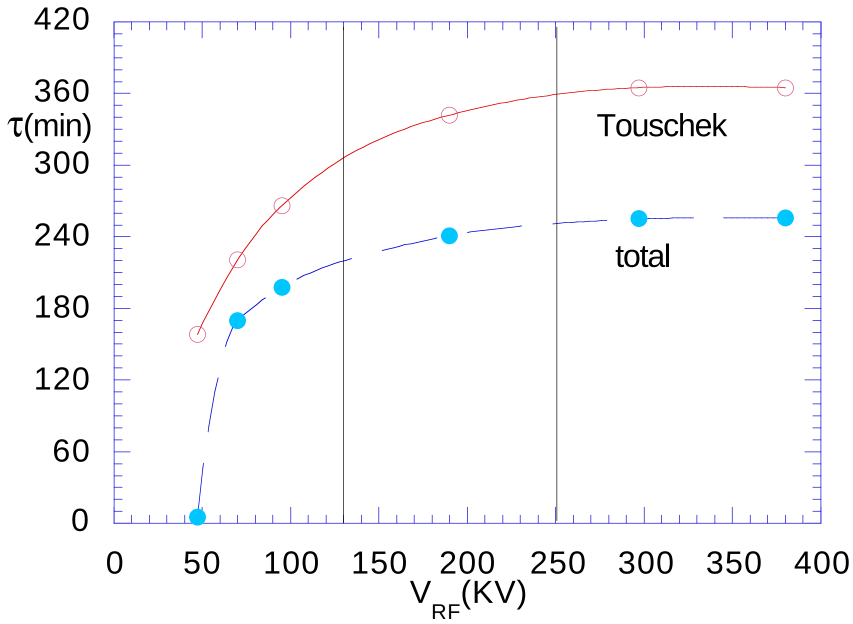


Fig. 5 -Touschek and total beam lifetimes as a function of the RF voltage.

The Touschek and total lifetimes, together with the gas scattering lifetime, are shown in **Table V** and in **Fig. 6** as a function of the horizontal aperture of the vacuum chamber, assumed constant along the ring, with the nominal value of the RF voltage.

Table V

A_x (cm)	τ_{Tou} (min)	τ_{sc} (min)	τ_{tot} (min)
2.15	74.9	797.	66.1
4.3	349.	1430.	245.
8.6	981.	1783.	475.

The beam lifetime is limited essentially by the vacuum chamber aperture, in fact increasing the physical aperture the beam lifetime still increases.

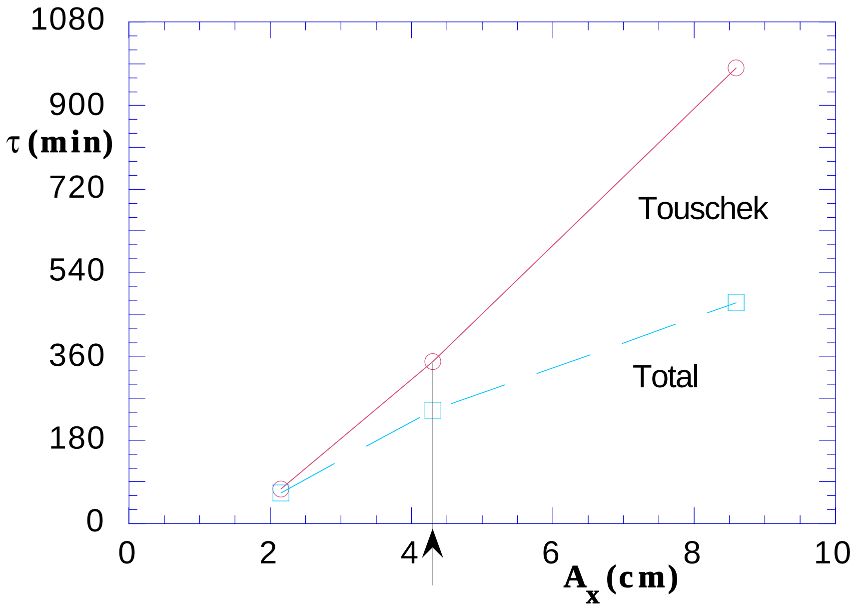


Fig. 6 - Touschek and total beam lifetime as a function of the horizontal aperture of the vacuum chamber, assumed constant along the ring.

The vertical aperture has been chosen in order to have a gas scattering beam lifetime much larger than the Touschek one. In **Table VI** Touschek, gas scattering and total lifetimes are shown as a function of the vertical aperture at the maximum β_y. The minimum required vacuum chamber aperture scales as the square root of the vertical β-function along the ring.

Table VI

A_y (cm)	τ_{Tou} (min)	τ_{sc} (min)	τ_{tot} (min)
2.3	342.	446.	176.
4.6	342.	1430.	241.
9.2	342.	3189.	266.

The chosen aperture is a balance between beam lifetime requirements and technical and economical constraints. The most stringent limit is the aperture of the low-β quadrupole triplet embedded into the experimental detector and constrained into an angular region of 9 degrees from the IP. For the other magnetic elements the feasibility of a large aperture with high field quality is an economical and technical problem at the same time. The final solution gives a beam lifetime of nearly four hours, which is quite good with respect to the foreseen injection time.

5. Dynamic Aperture requirements

The on energy dynamic aperture for the ideal machine, i.e. without multipole errors, is larger than the vacuum chamber aperture for all the DAΦNE lattices [5]. Off-energy the dynamic aperture becomes smaller. We have optimized the dynamic apertures at a relative energy deviation ε=.01 and at this energy they are of the same order of the vacuum chamber. When multipole errors in the magnets are introduced there is a reduction of the dynamic aperture. We want to estimate how much this reduction influences the beam lifetime and, mainly, what is the energy dependence of A_{DA} required to reach an acceptable beam lifetime.

The results of beam lifetime calculations for different dependencies of A_{DA} versus ε are presented below.

First of all we have assumed $A_{DA}(\varepsilon)$ constant in the energy interval between zero and the RF acceptance. The values of $A_{DA}(\varepsilon)$, in units of σ_x, are listed in **Table VII** with the corresponding values of the Touschek and total beam lifetimes calculated with the parameters of Table I.

Table VII

A_{DA}/σ_x				τ_{Tou} (min)	τ_{tot} (min)
ε = 0.	ε = .005	ε = .010	ε = .015		
11	11	11	11	342	241
10	10	10	10	327	233
8	8	8	8	214	169
6	6	6	6	119	87

A dynamic aperture as large as 11 σ_x, in the energy range between zero and the RF acceptance, corresponds to the physical aperture of the vacuum chamber. Aperture values smaller than 6σ_x have not been considered because, below this value, the quantum beam lifetime decreases very rapidly.

The beam lifetimes presented in **Figs. 7** and **8** and in **Table VIII**, **IX** show the effect of a reduction of the dynamic aperture at energies $\varepsilon = 10^{-2}$ and $\varepsilon = 5.10^{-3}$ respectively.

The beam lifetime is not very sensitive to the limits of the dynamic aperture at an energy ε = .01 while, at ε = .005, it decreases rapidly if the maximum stable oscillation amplitude A_{DA}(.005) is reduced.

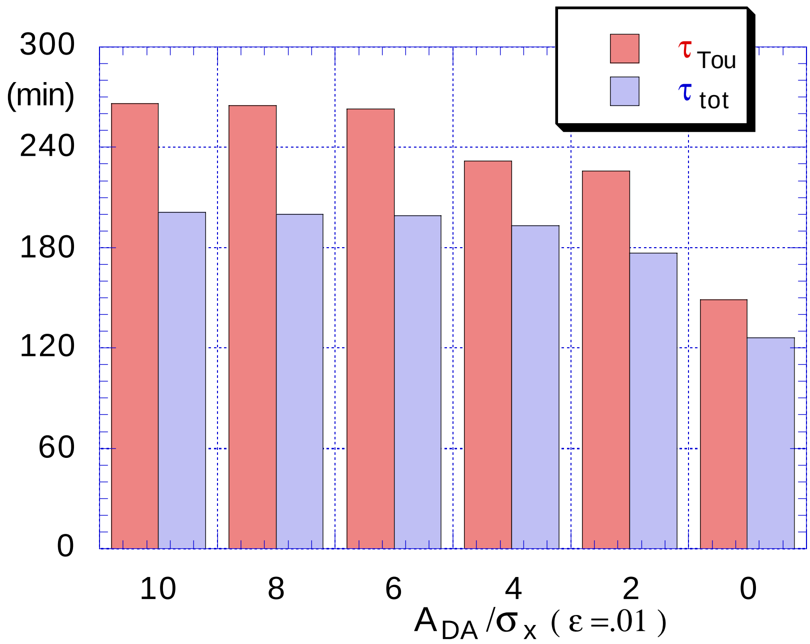


Fig. 7- Touschek and total beam lifetimes for different values
A_{DA}/σ_x(ε=.01) with: A_{DA}/σ_x(0.) =A_{DA}/σ_x(.005)=10.

Tab VIII

A_{DA}/σ_x				τ_{Tou} (min)	τ_{tot} (min)
ε = 0.	ε = .005	ε = .010	ε = .015		
10	10	10	0	266	201
10	10	8	0	265	200
10	10	6	0	263	199
10	10	4	0	252	193
10	10	2	0	226	177

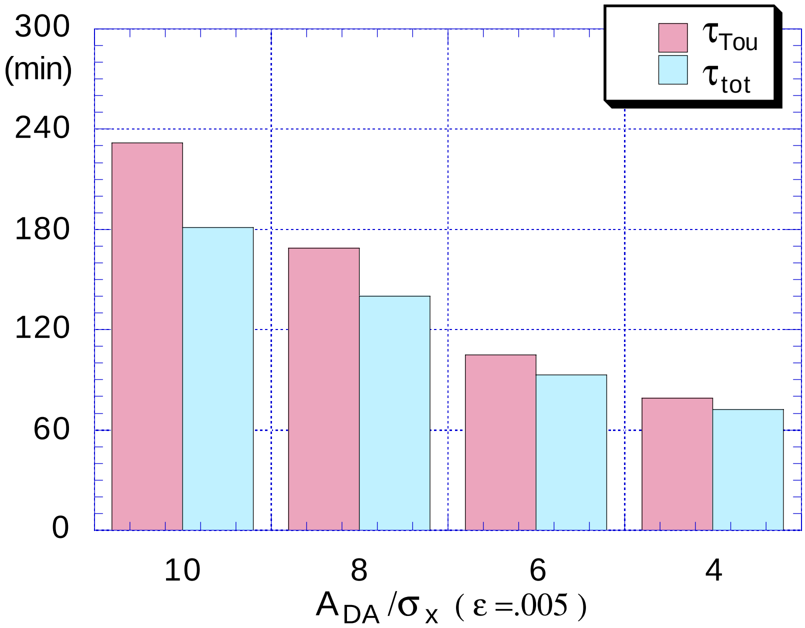


Fig. 8 - Touschek and total beam lifetimes for different values of $A_{DA}/\sigma_x(\varepsilon$**=.005**) with: $A_{DA}/\sigma_x(0.) = 10$, $A_{DA}/\sigma_x(.01) = 3$.

Tab IX

A_{DA}/σ_x				τ_{Tou} (min)	τ_{tot} (min)
ε = 0.	ε = .005	ε = .010	ε = .015		
10	10	3	0	232	181
10	8	3	0	169	140
10	6	3	0	105	92.8
10	4	3	0	79.0	72.1

In order to explain this effect the behaviour of H(s) and $\varepsilon_{lim}(s)$ along half the ring (the other half is symmetric) is shown respectively in **Figs. 9** and **10**. In Fig. 10 the continuous line is the vacuum chamber limit and the dashed one is the RF acceptance.

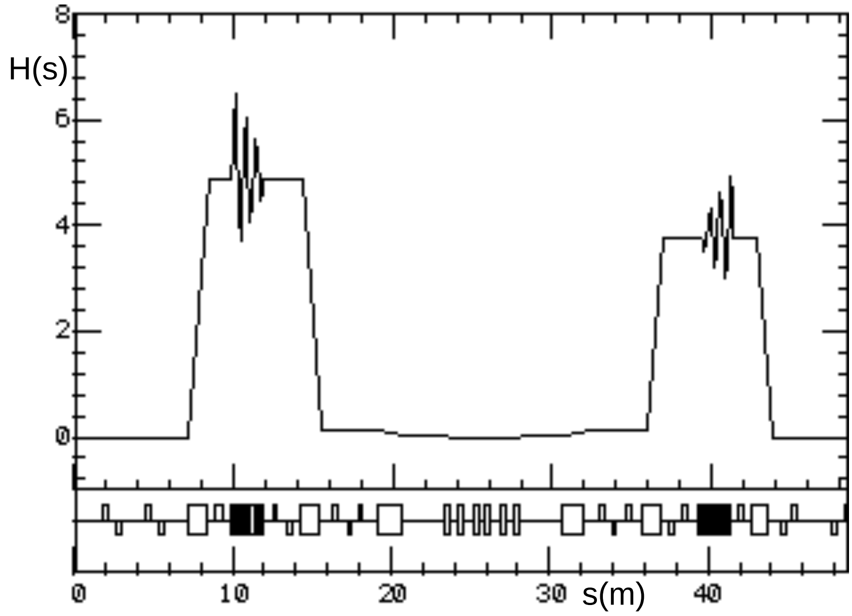


Fig. 9 - Behaviour of H(s) along half of the ring between the two symmetry points.

The ring can be divided into two regions; straight sections, where dispersion and H vanish, and arcs where, due to the high emittance, H is large. Particles undergoing a Touschek scattering in a straight section get only an energy variation and no betatron oscillation amplitude. The vacuum chamber energy limitation is larger than the design value of the RF acceptance.

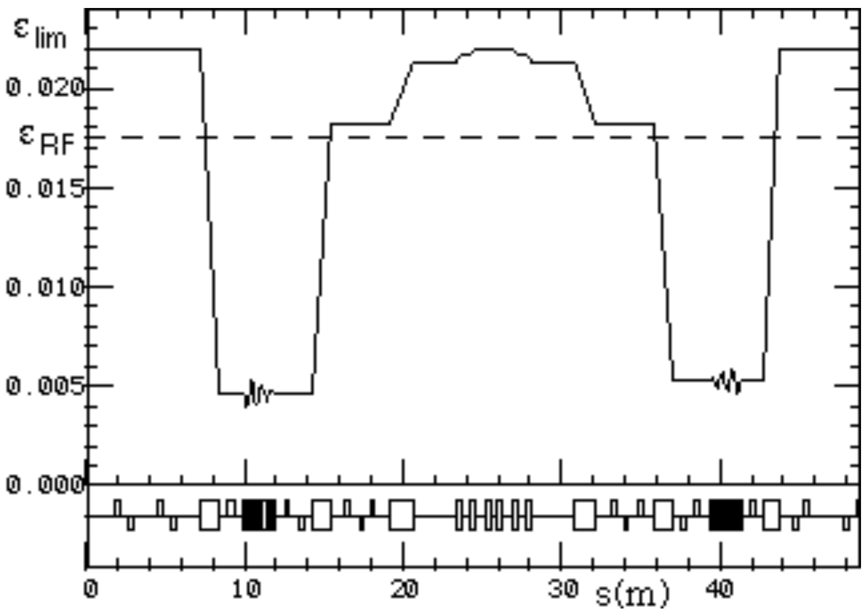


Fig. 10 - Behaviour of ε_{lim} along half of the ring between the two symmetry points. The continuous line is the vacuum chamber limit and the dashed one is the RF acceptance.

The particles which undergo a Touschek scattering in the arcs get a betatron oscillation amplitude proportional to the energy change and reach the vacuum chamber aperture for an energy $\varepsilon \sim 5\ 10^{-3}$. Therefore, below this energy, the amplitude of the dynamic aperture is important for the beam lifetime.

For energies $0. \leq \varepsilon \leq .005$, the dynamic aperture must be comparable with the physical aperture ($\sim 10\sigma_x$) to accept the large betatron amplitudes of the particles scattered in a dispersive region. For larger energies the dynamic aperture has to be large enough to accept particles which, due to the Touschek scattering, change only the energy and keep unperturbed the betatron amplitude. Therefore an aperture $A_{DA} \geq 3\sigma_x$ should accept most of those particles.

In Ref.[6] a detailed study of the dynamic aperture with systematic and random errors in all the magnetic elements is presented, performed for the D13 and D15 lattices. The introduction of multipole errors in the magnets gives a reduction of the dynamic apertures. In order to estimate the impact of this reduction, beam lifetime has been calculated for each error configuration presented in Ref.[6].

Dynamic apertures are evaluated in Ref.[6] for three energy values ε=0,+1%,-1%. To calculate the beam lifetime, the value of A_{DA} on energy and the smaller between the two off-energy has been used. Moreover, for each energy, the smaller horizontal limit of the dynamic aperture has been taken. This pessimistic assumption explains why in Ref.[6] sometimes different dynamic apertures give the same beam lifetime.

The Touschek and total beam lifetimes are listed in Ref.[6]; they are also shown in **Appendix B**, together with the shape of $A_{DA}(\varepsilon)$ and its values at 0 and 1% energies.

6. Conclusions

At the beam-beam limit the luminosity is proportional to the emittance of the machine. DAΦNE is designed with a large emittance, and therefore the physical aperture of the machine has to be very large. This implies major costs but also technical challenges. The more difficult problem to be solved is the design of the low-β quadrupoles which are the most limiting aperture. Another problem is the magnet design because a very good field quality is required on a large aperture.

The chosen aperture gives a good beam lifetime which means a high value for the integrated luminosity.

A lot of work has been done in the linear lattice design and in the optimization of the sextupole configuration in order to get a dynamic aperture as large as the physical aperture. In fact, for all the lattices described in ref.[5], the dynamic aperture for the ideal machine, i.e. without multipole errors in the magnets, is equal to or larger than the physical aperture either on energy and for an energy deviation as large as ±1%.

When multipole errors in the magnetic elements are introduced [6] there is a small reduction of the dynamic apertures, in particular for off energy particles, but with the error values calculated by the magnet codes for our design the reduction is not harmful to the beam lifetime. Taking into account the foreseen injection time the average luminosity can be very near to its peak value.

The coupling factor is a parameter which also influences the luminosity. For DAΦNE it has been chosen to operate with very flat beams (κ=.01) [4]; therefore the vertical beam size is very small. The vertical vacuum chamber aperture has been chosen to yield a gas scattering beam lifetime much larger than the Touschek one and it is $A_y = 10\sigma_y$ (full coupling). As a consequence the DAΦNE vertical aperture is quite large as compared to the beam size in the operating conditions ($A_y = 70\sigma_y$ for κ=.01).

If we want to change the operating conditions, increasing the coupling factor, we still have a good safety margin on the quantum lifetime (for κ=.1 it is $A_y = 20\sigma_y$).

The dynamic aperture in the vertical plane is always larger than the vacuum chamber.

We are not able to estimate the aperture requirements due to the beam-beam interaction but, for a flat beam collider, we expect that any enlargement or blow-up of the beam due to the beam-beam effect is in the vertical plane. By choosing a large vacuum chamber in the vertical plane beam losses due to vertical blow-up should be avoided.

References

[1] M.E. Biagini, S. Guiducci, M.R. Masullo, G. Vignola: "DAΦNE lattice update", DAΦNE Technical Note L-4, 12/2/1991.

[2] G. Vignola: "Leda computer code", unpublished.

[3] H. Bruck: "Accelerateurs circulaires de particules", presses Universitaires de France, 1966.

[4] S. Bartalucci, M. Bassetti, M.E. Biagini, C. Biscari, S. Guiducci, M.R. Masullo, L. Palumbo, B. Spataro, G. Vignola: "DAΦNE design criteria", DAΦNE Techn. Note G-2, 11/12/1990.

[5] M.E. Biagini, C. Biscari, S. Guiducci, J. Lu, M.R. Masullo, G. Vignola: "Review of DAΦNE lattices", DAΦNE Technical Note L-9,10/12/1993.

[6] M.E. Biagini, M.R. Masullo: "Dynamic aperture with systematic and random multipole errors in DAΦNE", DAΦNE technical note L-10,10/24/1993.

Appendix A

Example of input data for the LEDA code

```
 Dafne13
# OF ELEMENTS  ----  # OF FAMILIES  -----  # OF CONDITIONS    (FREE FORMAT)
 463   0   2
# OF PERIODS  -----  NM1,NM2.....NM10( MATCHING POINTS)     (11 DATA F.F.)
  1    57   97 105  107  0 0   0   0  0  0  0  0  0  0  0  0
ENERGY(MeV) -----ZAPFILE(0=NO,1=YES)-----ACCURACY             (3 DATA F.F.)
  510.                    0.     1.D+09
PAR FLAG (0/1) -- SYMFLAG(0=NO,1=YES) --SCREENFLAG(0/1) -- DISPERS MULTIPLIER
  1                  1                      1                  1.
IDENTIFIERS OF VARIABLES (MUST BE = # OF CONDITIONS)          (FREE FORMAT)
  105  103   101 107   10 12   20 24 93
   QxW          QzW
  5.18         6.15
CORRECTED CHROMATICITIES(CRXCOR,CRZCOR)
   0      0
FIT CHROMATICITIES and EMITTANCE ( CRXW,CRZW,EMW  --- when used)
-.3 -3.66  1.D-06
RF RELATED PARAMETERS ( MUST BE OK WHEN  PAR-FLAG=1)
 I(mA) - COUPL - VRF(KV) - RF HARMONIC - HOR APERT(m) - VERT AP(m) - Z/n(Ohm)
  43.75    .01      70.       120           .043         .046       0.5
ADDITIONAL CONSTANTS:
BX0         BZ0       ETA0W
 8.761      2.358     2.379E-6
BX1        BZ1        ETA1W                 BX2       BZ2      ETA2w
2.2        1.198157      -.0                    -1.19      0.        0.
BX3        BZ3        ETA3W                 BX4       BZ4      ETA4w
 -.66      -.66          1                   1          1        1
BX5        BZ5        ETA5W                 BX6       BZ6      ETA6w
 1          1           1                   1          1        1
BetaDA     etaDA
 4.5        0.0
XDA                                        DPDA
 .021  .021  .021  .021  .021 .0           0.  .005  .01  .015  .02  .025
ALCOST(# 0)     FRFW
  1.22e-2        0
xxxx
xxxxx
xxxx
 CELL INPUT : FORMAT(3I4-5G(10.6)   (FOLLOW THE FIELD INDICATOR)
iiiijjjjiiiiLLLLLLLLLLKKKKKKKKKKRRRRRRRRRRFFFFFFFFFFAAAAAAAAAA
   1        11.87759151                                       .032
   2        2 .3       2.8178242
   3        1 .45
   4        3 .3       3.2005238
   5        1 1.64
   6        2 .30      4.6458343
   7        1 .48
   8        3 .30      1.5252287
   9        1 .2
  10        7 0.       0.
  11        1 1.35
  12        6          24.75     1.4005635
  13        41.21                1.4005635                   .047
  14        6          24.75     1.4005635
  15        1 .6                                             .05
  16        2 .30      1.937606                              .05
```

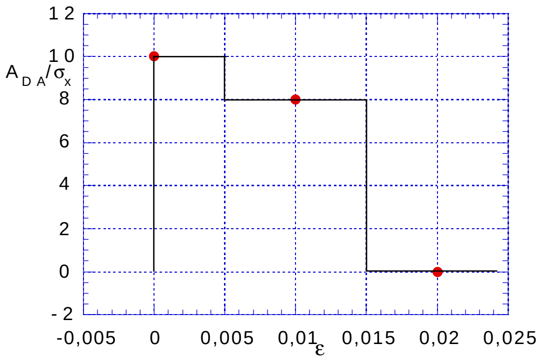
Example of the output of the LEDA code with the beam lifetimes.

BEAM PAR & dN/dt FOR T=293K - P=1nTorr - Z(biatomic)=8 :

REV. FREQUENCY (MHZ)	0.306880414D+01	
HARMONIC NUMBER	0.120000000D+03	
RF FREQUENCY (MHZ)	0.368256496D+03	
VRF(KV)	0.190000000D+03	
ENERGY (MEV)	0.510000000D+03	
U0 (KeV)	0.932172088D+01	
MOM. COMPACTION	0.588522813D-02	
F SYNC.(KHz)	0.198463920D+02	
RF ACCEPTANCE	0.176168346D-01	
NAT.BUNCH LENGTH(cm)	0.560590225D+00	
NAT. ENERGY SPREAD	0.396207735D-03	
# ELECTRONS/BUNCH	0.889910589D+11	
AV.CURRENT/BUNCH(mA)	0.437500000D+02	
PEAK CURRENT/BUNCH(A)	0.304154422D+03	
Z/n (Ohm)	0.500000000D+00	
PEAK CURRENT M.W. Thres. (A)	0.592092067D+01	
ANOMALOUS BUNCH LENGTHENING QUANTITIES :		
AN. BUNCH LENGTH(cm)	0.300000000D+01	
REL. R.M.S. ENERGY-SPREAD	0.150000000D-02	
PEAK CURRENT/BUNCH(A)	0.568353319D+02	
EMITTANCE (mm-mrad)	0.103184755D+01	
EMITTANCE COUPL.	0.100000000D-01	
HOR. HALF-APERTURE (cm)	0.430000000D+01	
VER. HALF-APERTURE (cm)	0.460000000D+01	
LIMITS D.A. vs ENERGY	0.029	0.000
	0.025	0.005
	0.021	0.010
	0.000	0.015
	0.000	0.020
	0.000	0.025
beta DA, eta DA	4.500	0.000
DP/Pmax @ 140 EL.	0.220006663D-01	
QUANTUM LIFE E.O.(hrs)-SANDS	0.320060062D+23	
QUANTUM LIFE R.O.(hrs)-SANDS>>>	0.938200759D+31	
LIFETIME GB (min)	0.235126007D+04	
LIFETIME SC (min)	0.142979767D+04	
LIFETIME GBe (min)	0.144692927D+05	
LIFETIME SCe (min)	0.322712093D+05	
TOUSCHEK (min)	0.276061309D+03	
LIFETIME TOT.(min)	0.206305244D+03	

Appendix B

Beam lifetimes as a function of $A_{DA}(\varepsilon)$
with the configuration shown in the figure.



Behaviour of A_{DA}/σ_x versus ε used to calculate the beam lifetimes cor ing to the dynamic apertures with multipole errors.

Table

A_{DA}/σ_x		τ_{Tou} (min)	τ_{tot} (min)
ε = 0.	ε = .010		
10.	10.	301.	220.
10.	9.	293.	216.
10.	8.	290.	214.
10.	7.	287.	212.
10.	6	283.	210.
10.	4.	260.	197.
8.	7.	201.	161.
8.	5.	195.	157.
8.	4.	188.	153.
7.	7.	157.	132.
7.	6.	156.	131.
7.	5.	153.	129.
7.	4.	149.	126.
7.	3.	140.	120.
7.	1.	113.	99.5
6.	5.	114.	85.
6.	4.	112.	83.4
6.	3.	107.	80.7
6.	1.	90.6	71.0