DAΦNE TECHNICAL NOTE

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ESTIMATE OF HOURGLASS EFFECT IN DAΦNE

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1. Introduction

The DAΦNE luminosity is measured both by a single bremsstrahlung luminosity monitor [1] and by the KLOE detector [2]. Moreover the luminosity is estimated with the usual luminosity formula on the basis of the beam sizes measured at the synchrotron light monitor (SLM), translated at the interaction point (IP) by using the beta-functions from the machine model. Once calibrated, the luminosities measured by the luminosity monitor and by the detector are in a good agreement and closely follow each other during experimental runs while the colliding beam currents decrease. On the other hand it has been observed that the analytical formula agrees well with the measured luminosities at low beam currents, but it substantially overestimates the luminosity for high beam currents. The disagreement is higher for higher bunch currents and it can reach almost a factor of 2 in the nominal operating conditions.
There are a few possible reasons that can explain such a disagreement in multibunch collisions:

1) the synchronous phase spread along the bunch train grows with the beam currents. This means that different bunches collide at slightly different longitudinal position and the resulting luminosity no longer scales exactly with the number of bunches;
2) the hourglass effect [3], which is a limitation with the present KLOE optics, with $\beta_y^* = 1.9$ cm and measured bunch lengths as long as 2.5 cm for a 10 mA bunch current, becomes more pronounced for higher bunch currents;
3) a further geometrical reduction factor is due to the horizontal crossing angle.

A study of the hourglass effect on DAΦNE luminosity has been done in order to:

1) numerically estimate the luminosity loss due to the hourglass in the current working conditions;
2) evaluate the contribution of the finite crossing angle;
3) compare the hourglass effect for different bunch distributions. This is important since the bunch shape gets more and more parabolic for higher bunch currents (especially for the electron ring, see [4]);
4) possibly find an optimum value of the beta function at the IP;
5) calculate the beam-beam tune shift changes due to the hourglass effect.

In Section 1 the basic formulae and techniques necessary to estimate the hourglass effect are briefly described, and a comparison with existing analytical formulae is shown. In Section 2 luminosity variation plots as a function of bunch length, crossing angle, and betas at IP are presented. Comparison between three typical distributions, Gaussian, parabolic and rectangular, is also shown. The beam-beam tune shift changes due to the hourglass effect are discussed in Section 3. The results of 6D numerical simulations are presented in Section 4.

2. Luminosity calculation

In order to compute the luminosity with hourglass and crossing angle, different methods have been used. First the luminosity has been computed for gaussian beams with Hirata's analytical formula [5] which takes into account the effect of the finite horizontal crossing angle:

$$\begin{gathered}\frac{L}{L_o} = \sqrt{\frac{2}{\pi}}\, a e^{b} K_o(b) \\ a = \frac{\sigma_y^*}{\sqrt{2}\,\sigma_l\,\sigma_{py}^*}, \quad b = a^2\left[1+\left(\frac{\sigma_l}{\sigma_x^*} tg\frac{\phi}{2}\right)^2\right]\end{gathered} \tag{1}$$

where ϕ is the total crossing angle, σ_x^*, σ_y^* are the transverse beam sizes at IP, σ_l is the bunch length, σ_{py}^* is the vertical beam divergence at IP and L_0 is the luminosity without neither hourglass nor crossing angle and K_o is the modified Bessel function. The parameters used for the calculation are listed in Table 1.

Table 1 – Beam parameters

ε_x (mm mrad)	0.36
β_x^* (cm)	170
β_y^* (cm)	1.9
$\phi/2$ (mrad)	16.5
κ (%)	0.3

The bunch length values have been extrapolated from the measurements taken with positive and negative α_c lattices in both rings [6], and shown in Fig. 1.
The electron bunch length was measured at 165 kV RF cavity voltage, while the positron one was measured at 110 kV. Table 2 summarizes the measured values for zero current and a typical bunch current of 10 mA, both scaled at 165 kV.

Table 2 — Measured bunch lengths

Conditions	σ_l (e^-) [cm]	σ_l (e^+) [cm]
10 mA, α_c >0	2.4	1.65
10 mA, α_c <0	1.5	1.
0 mA	1.	1.

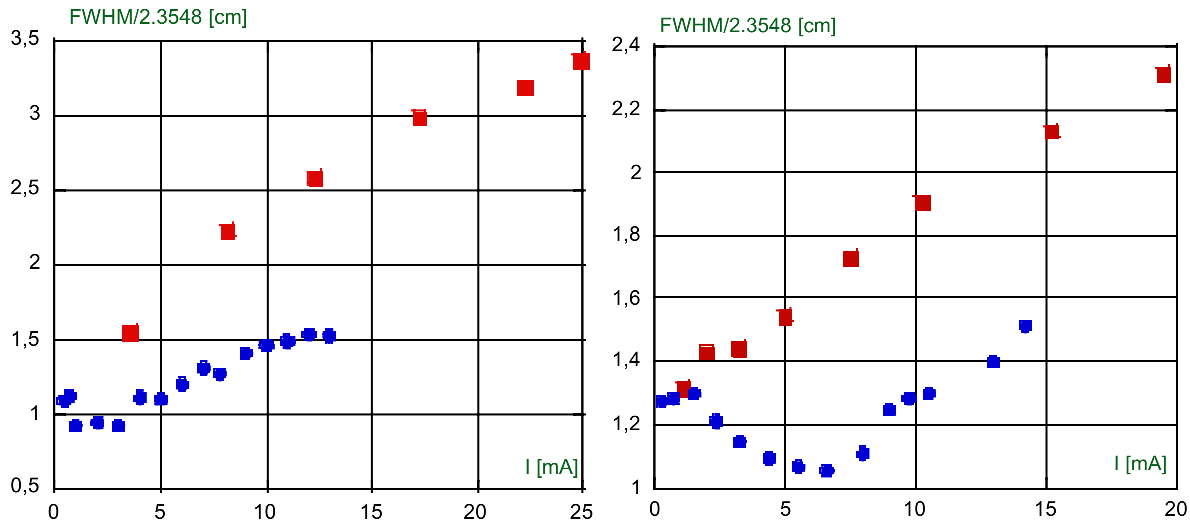

Fig. 1 — Bunch length measurements for electrons (left) and positrons (right) for positive α_c (upper curve) and negative α_c (lower curve)

The luminosity reduction factor L/L_0 due to hourglass and crossing angle, computed with formula (1) as a function of the bunch length, is shown in Fig. 2. If we consider an average bunch length of 2 cm at 10 ma/bunch current, the luminosity reduction is 20%, while it is only 8% for a 1 cm long bunch.

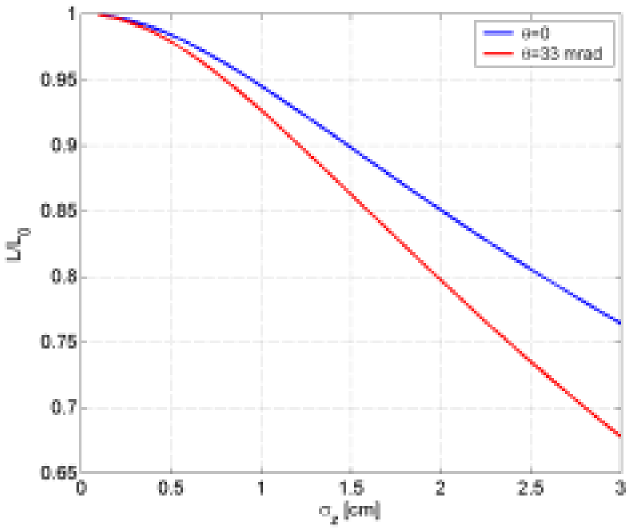

Fig. 2 — Luminosity reduction factor as a function of the bunch length (Hirata s formula)

In order to investigate the effect of different longitudinal bunch profiles the following luminosity formula has been used:

$$L = 2\cos^2(\phi)\int \rho_x(x_1)\rho_x(x_2)\rho_y^2(y)\rho_{s1}(s_1 - ct)\rho_{s2}(s_2 + ct)dxdydsdct \qquad (2)$$

where ρ_x, ρ_y are the transverse beam distributions (gaussians with equal rms for the two beams), and ρ_{s1}, ρ_{s2} the longitudinal beam distributions, which might have different shape. In formula (2) the crossing angle effect is taken into account by the factor $2\cos^2\phi$ and by performing a change of variables for x and s (horizontal and longitudinal position) for the two colliding beams:

$$\begin{aligned} s_1 &= s\cos\phi + x\sin\phi \\ s_2 &= s\cos\phi - x\sin\phi \\ x_1 &= -s\sin\phi + x\cos\phi \\ x_2 &= s\sin\phi + x\cos\phi \end{aligned} \tag{3}$$

while the hourglass effect by considering a vertical beam size changing with the longitudinal position *s*:

$$\sigma_y = \sigma_y \sqrt{1+\left(\frac{s}{\beta_y^*}\right)^2} \tag{4}$$

Two codes have been written to compute equation (2), one performing a numerical integration and the other integration by an optimized Monte Carlo (MC) method. In the first case the domain of integration is discretized with a cubic mesh. In the second the integral is performed by randomly extracting a large number (N_{est}) of times in the integration interval, optimizing by extracting from gaussian distributions g_i and weighting by ρ_i/g_i to compensate for the introduced bias. This method is simpler and faster with respect to a numerical integration procedure and for equal computing time it is also more precise, provided N_{est} is large, since there are no problems of step definition or borders.
To estimate the contribution to the luminosity of a non-gaussian beam profile, a comparison between different bunch shapes, like gaussian, parabolic and rectangular, has been performed. The bunch profiles considered, with the same rms value, are plotted in Fig. 3.

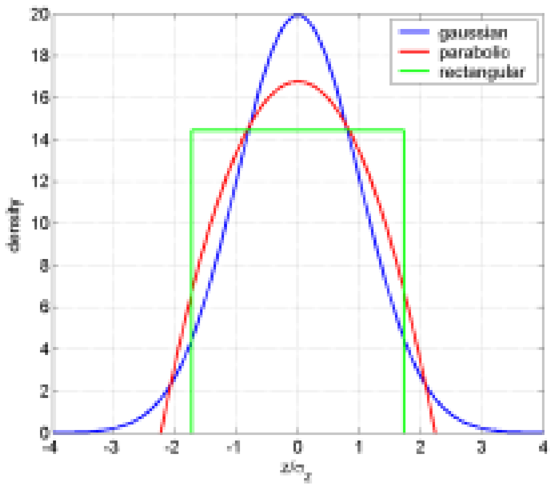

Fig. 3 — Different bunch profiles with the same rms

The numerical integration and MC method results agree (see Appendix A). The effect of a non-gaussian bunch profile on the luminosity reduction factor is few percent, as shown in Fig. 4, where L/L_0 with and without crossing angle is plotted as a function of the rms bunch length and considering different bunch shapes (parabolic+gaussian meaning that a gaussian beam was colliding with a parabolic one).

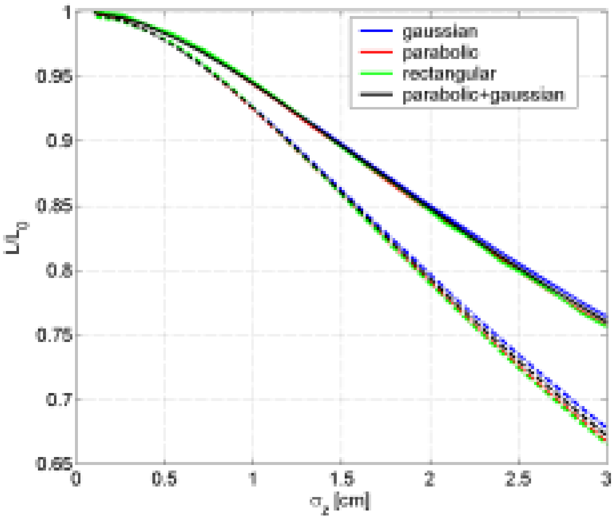

Fig. 4 —Luminosity reduction factor as a function of the bunch length for different bunch shapes, for zero crossing (upper curves) and nominal crossing angle

3. Tune shifts and best operating parameters study

In order to study the possibility of increasing the luminosity by decreasing β_y* and/or the bunch length, Hirata s formula and numerical integrations have been applied to compute the luminosity behavior in presence of hourglass effect and crossing angle. For a nominal 10 mA bunch current the luminosity has been computed using formula (1), as a function of β_y* for different bunch lengths but equal bunches, as shown in Fig. 5.

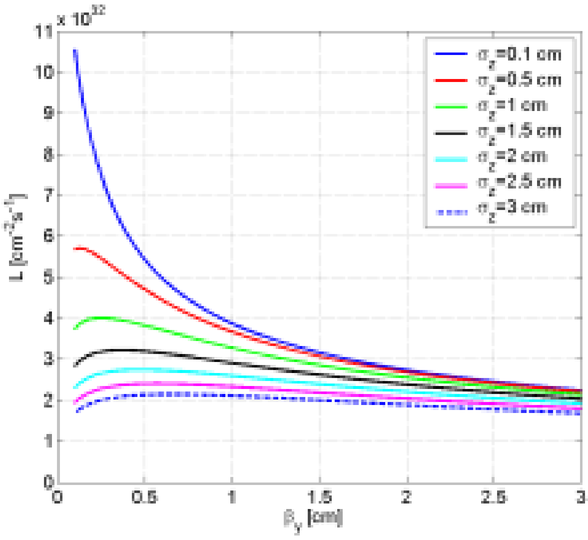

Fig. 5 — Luminosity as a function of β_y^* for different bunch lengths (Hirata s formula)

The case of two gaussian beams with different rms bunch lengths (2.4 and 1.65 cm) as actually measured at 10 mA, and computed with the MC method, is plotted in Fig. 6.

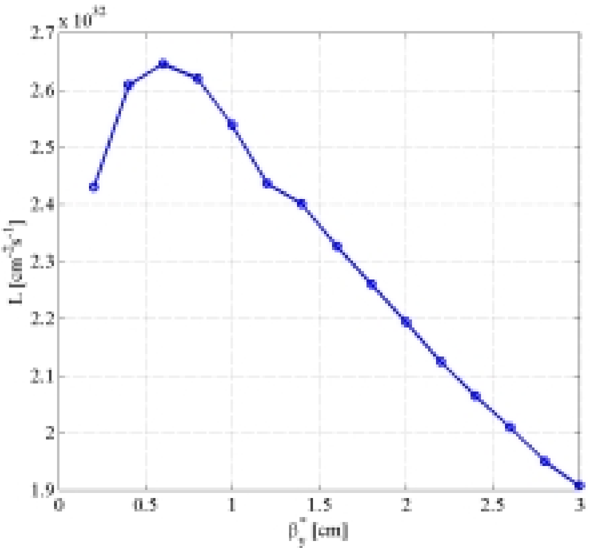

Fig. 6 — Luminosity as a function of β_y^* in the case of two gaussian beams with different rms bunch lengths (2.4 and 1.65 cm) as actually measured at 10 mA

The hourglass effect on the integrated e^+ vertical beam-beam kick as a function of the longitudinal position z has also been computed, following the formula [7]:

$$\xi_{y+}(z)=\frac{1}{4\pi}\int ds\beta_{y+}(s)\Delta K_{y+}(s)=\frac{r_0 N_-}{\pi\sqrt{2\pi}\gamma_+\sigma_{z-}}\int ds\frac{\beta_{y+}(s)e^{-(2s-z)^2/2\sigma_{z-}^2}}{\sigma_{y-}(s)[\sigma_{x-}(s)+\sigma_{y-}(s)]} \tag{5}$$

where + and - refer to the two colliding beams and N is the number of particles/bunch. In eq. (5) the crossing angle is not taken into account. The corresponding tune shift as a function of z is given by[8]:

$$\cos(2\pi(Q_{0y}+\Delta Q_y))=\cos(2\pi Q_{0y})-2\pi\xi_y\sin(2\pi Q_{0y}) \tag{6}$$

where Q_{oy} is the nominal fractional tune. Fig. 7 shows the vertical beam-beam kick and tune shift, computed by numerical integration along the bunch profile, as a function of z, assuming $Q_{oy} = 0.165$.

It is clear from this plot how particles in the bunch tails experience a larger tune shift. The difference between ξ_y and ΔQ_{oy} is few percent at this working point. For this reason in the following analysis we have used the ξ_y parameter only.

The average vertical beam-beam kick $\bar{\xi}_y$ as a function of β_y^* and for different bunch lengths (two gaussian bunches with equal bunch lengths) is plotted in Fig. 8.

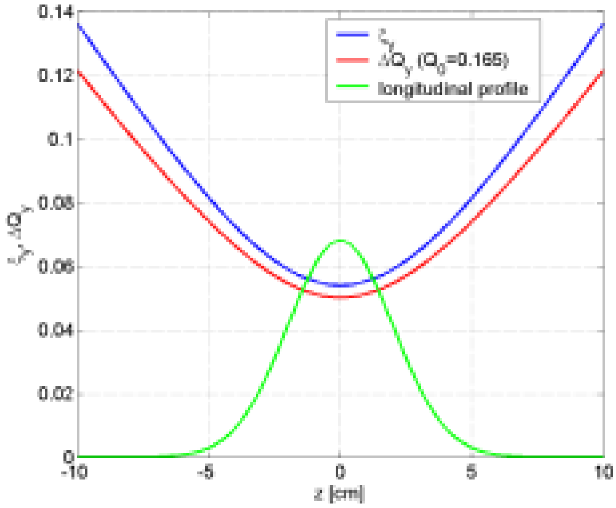

Fig. 7 — Longitudinal bunch profile (green) with vertical tune shift (red) and beam-beam kick (blue) as a function of z.

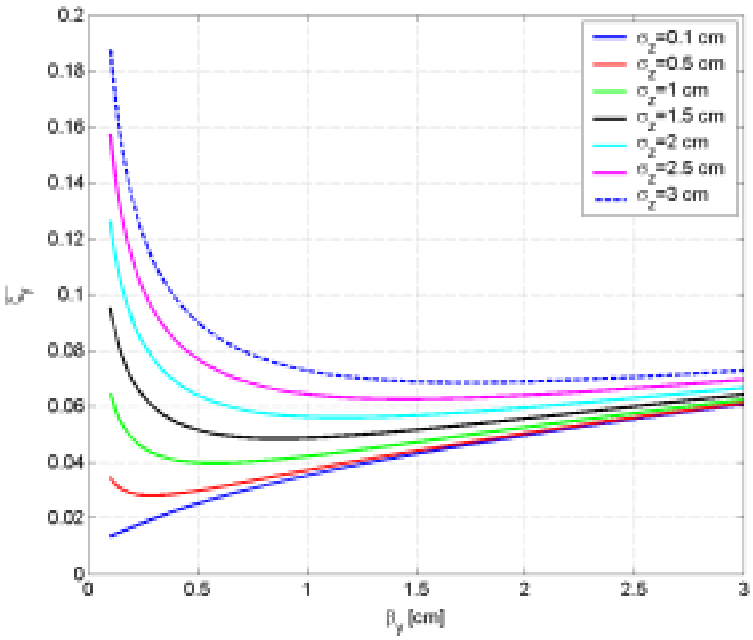

Fig. 8 — Average vertical beam-beam kick as a function of β_y^* for different bunch lengths

To evaluate the possible gain in peak luminosity coming from shorter bunches and/or lower β_y^*, the ratio of the luminosity and vertical beam-beam kick, normalized to the actual tune shift limit as computed from the present peak luminosity and beam parameters ($\overline{\xi}_y^{\max} = 0.031$ for $\sigma_l = 2.5$ cm at 10 mA/bunch) is plotted in Fig. 9. We define as normalized luminosity the quantity:

$$L_{norm} = \left(\frac{L}{\overline{\xi}_y} \right) \overline{\xi}_y^{\max} \tag{7}$$

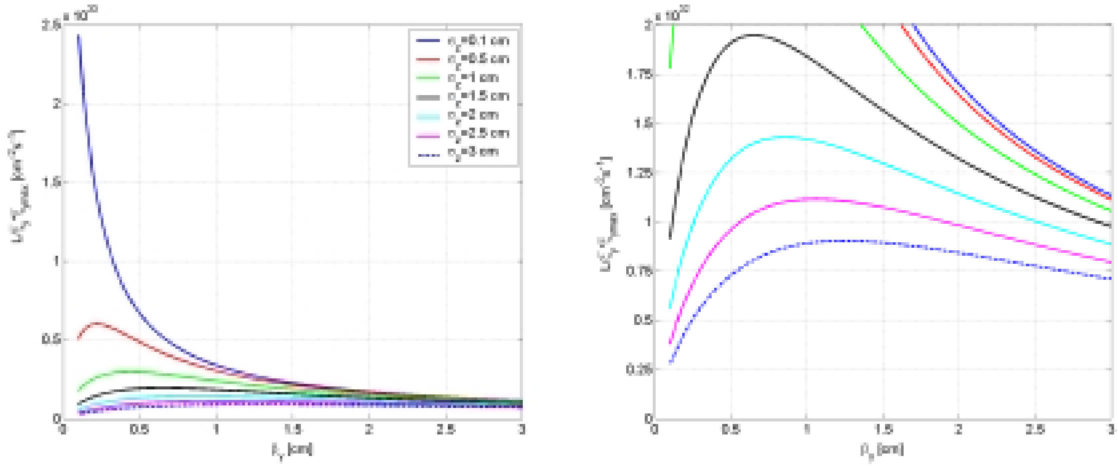

Fig. 9 — Luminosity normalized to beam-beam kick shift as a function of β_y^* for different bunch lengths (left) and zoom between L=0 and $L=2\cdot10^{32}$.

It can be seen that with the positive α_c (longer bunch length), reducing β_y^* to 1 cm in principle a factor of 20% in luminosity can be gained. With a negative α_c (shorter bunch length) a 25% gain factor is still possible with the present value of β_y^*, while reducing β_y^* to 1 cm this factor could theoretically reach 80%.

4. Beam-beam simulations

In order to take into account the nonlinear beam-beam kick in addition to the geometric luminosity reduction factor 6D beam-beam simulations with the weak-strong code BBC [9] has been performed.

The simulations have been carried out for both negative and positive momentum compaction factors. The bunch length was taken to be 2.25 cm for the positive momentum compaction and 1.25 cm for the negative one (for simplicity, the bunch length was assumed equal for positrons and electrons), assuming a bunch current of 10 mA. The results are summarized in Fig. 10. The simulations were performed for both the present working point of the positron ring (0.115, 0.195, curves c) and d)) and the optimum working point (0.057, 0.097, curves a) and b)) that seems to provide highest luminosity in the tune area near the integer tunes.

The curves have all a behaviour similar to that predicted by the geometric reduction factor (see Fig. 5). However, the luminosity maxima are shifted towards to higher beta function values. For all the curves, the maxima are close to the value $\beta_y^* = \sigma_z/\sqrt{2}$, that is the optimum ratio predicted also for round colliding beams [10]. This value corresponds to the minimum value of the vertical beam-beam kick (see Fig. 8) discussed in the previous paragraph. According to the simulations, for the present working point a reduction of β_y^* gives almost no gain in luminosity. A gain can instead be achieved either by applying the negative momentum compaction together with the beta function reduction or by changing the working point. The best choice would be a simultaneous action: application of a lattice with negative momentum compaction factor and working point shift.

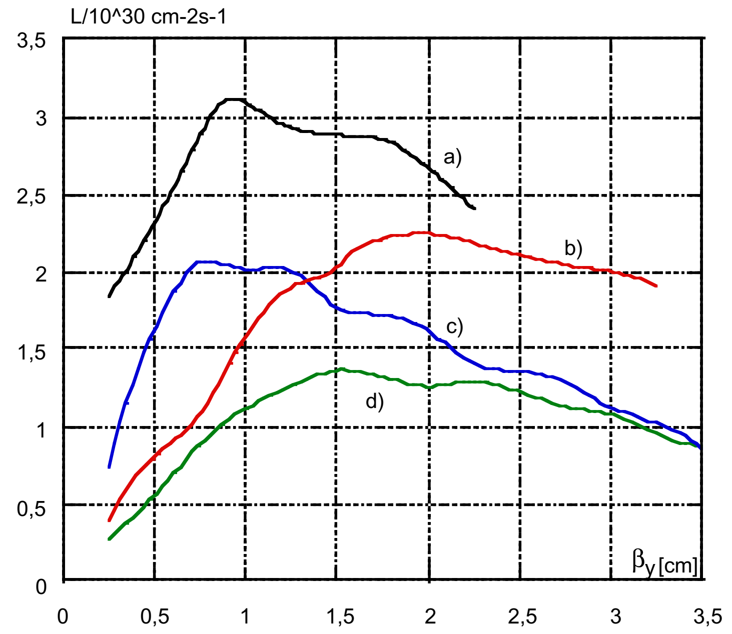

Fig.10 - Luminosity as a function of the vertical beta at IP: a) $\alpha_c < 0$, (0.057, 0.0957); b) $\alpha_c > 0$, (0.057, 0.0957); c) $\alpha_c < 0$, (0.115, 0.195); d) $\alpha_c > 0$, (0.115, 0.195)

5. Conclusions

From this study the following conclusions can be drawn:

1. in the worst condition, even assuming 15 mA per bunch in both electron and positron bunches, the geometric luminosity reduction factor due to the hourglass effect does not exceed 30%;
2. the contribution of the crossing angle to the luminosity reduction is estimated to be less that 10%;
3. for different distributions, having the same rms size, the luminosity reduction due to the geometric hourglass effect changes only within a few percent;
4. considering 1, 2 and 3, it can be concluded that the disagreement between the luminosity measured by the KLOE detector and that calculated using the well known luminosity formula cannot be explained by the hourglass effect alone. Presumably, the discrepancy is a combined effect of hourglass, synchronous phase spread, orbit variation with current, etc.;
5. with the present working conditions (given β_y^* and tunes) the luminosity gain can hardly be achieved by further reducing β_y^*. This is explained by the fact that the present $\beta_y^* = 1.9$ cm is already close to the optimum value (the flat maximum in the luminosity behavior being located at 1.5 cm). Besides, it has to be considered that a further reduction of β_y^* would lead to vertical chromaticity increase with all its unwanted consequences;

6. with a negative momentum compaction a luminosity increase of about 20% can be achieved with the present β_y^*. In order to exploit the advantages of a lattice with the negative momentum compaction, β_y^* should be decreased down to 1 cm. In this case, according to the numerical simulations, the luminosity gain can be as high as 70% (see Fig. 10);
7. it is worthwhile to explore tune areas close to the integer tunes which are a potential source for further luminosity increase. This conclusion is based not only on the simulation results presented in Fig. 10, but also on a common experience of many lepton colliders (ADONE, VEPP-2M and B-Factories for half-integer tunes).

References

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Appendix A

To compare the two different numerical codes, the following calculations, assuming gaussian beams and comparing the numerical results with the Hirata's formula (1), have been performed:

a) L/L_0 as a function of the bunch length using the parameters of Table 1 with and without crossing angle (Fig.A1);

b) L/L_0 as a function of β_y^* using the parameters of Table 1 with σ_z=2.5cm with and without crossing angle (Fig.A2);

c) Average tune shifts as a function of β_y^* using the parameters of Table 1 with σ_z=2.5cm and $I_{single\ bunch}$=10mA (Fig.A3);

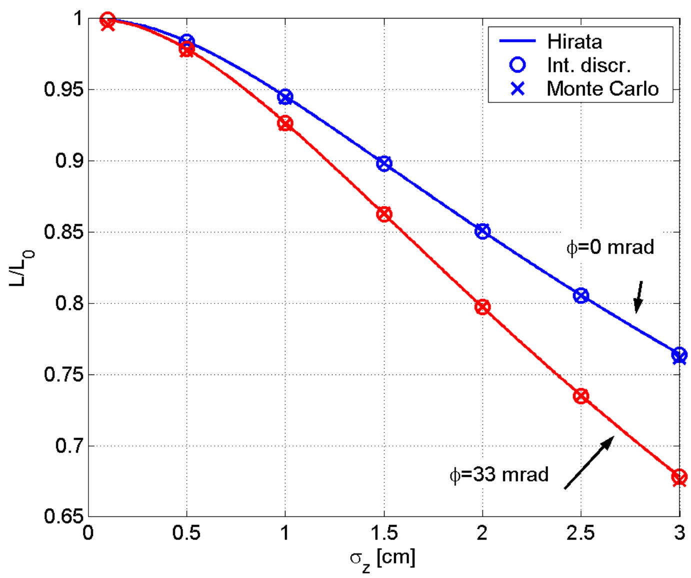

Fig A1- Comparison between the two numerical codes considering gaussian beams with different bunch lengths and using the parameters of Table 1

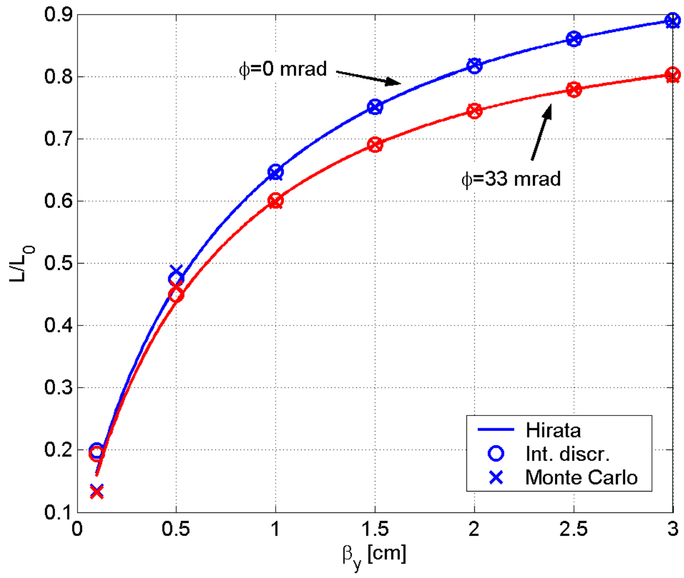

Fig A2- Comparison between the two numerical codes assuming gaussian beams with σ_z=2.5cm and using the parameters of Table 1

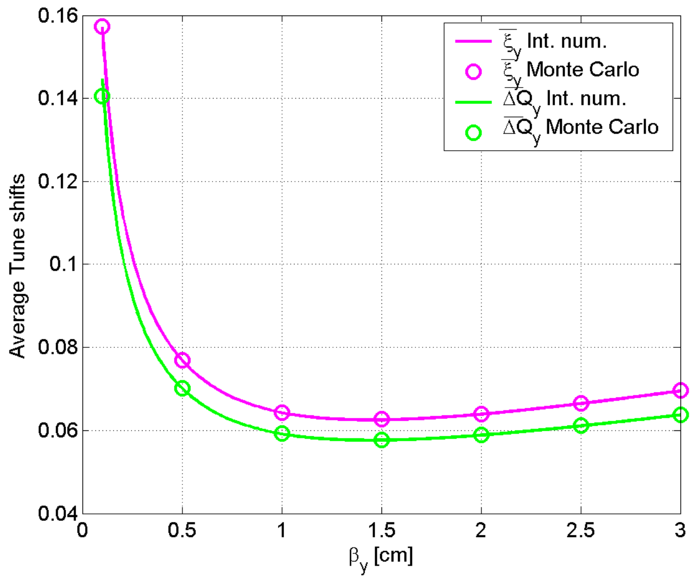

Fig A3- Comparison between the two numerical codes assuming gaussian beams with σ_z=2.5cm and $I_{single\ bunch}$=10 mA