DAΦNE TECHNICAL NOTE

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TRANSVERSE MODE COUPLING INSTABILITY IN THE DAΦNE ACCUMULATOR RING

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Abstract

For a single bunch storage ring the following single bunch transverse instabilities can be harmful: head-tail instability, resistive wall instability and transverse mode coupling instability (TMCI).

The first two instabilities can be easily cured. Namely, the head-tail instability is avoided by adjusting the machine chromaticity to a slightly positive value, while the resistive wall instability can be completely eliminated by choosing machine tunes above integers [1].

The latter instability, the TMCI, is the most dangerous one which can put a severe limit to the accumulated bunch current. Practically, there is no reliable means to cure the instability if enough care is not taken to reduce the transverse machine impedance.

In this paper we estimate the transverse beam impedance of the DAΦNE accumulator ring by measuring the shift of transverse tunes and betatron sidebands versus bunch current and evaluate the threshold of the TMCI.

Introduction

The shift of the coherent mode frequencies away from their low intensity values is given by well-known Sacherer's formula [2,3]:

$$\Delta\omega_{c_m} = j\frac{1}{1+|m|}\frac{Ic^2}{4\pi f_0 Q(E/e)L} Z_m^{eff} \qquad (1)$$

where m is the mode number ($m = 0; \pm1; \pm2,..$); f_0 the revolution frequency; I the average bunch current; Q the betatron tune; E the machine energy; L the full bunch length. The effective transverse impedance Z_m^{eff} is calculated as a machine transverse broad band impedance Z_T averaged over a coherent mode power spectrum h_m:

$$Z_m^{eff} = \frac{\sum_p Z_T(\omega_p)h_m(\omega_p - \omega_\xi)}{\sum_p h_m(\omega_p - \omega_\xi)} . \qquad (2)$$

Here the summation is performed over the mode spectrum lines at frequencies:

$$\omega_p = (p + \Delta Q)\omega_0 + m\omega_s\,; \quad -\infty < p < +\infty \tag{3}$$

with ΔQ being the fractional part of the machine tune; ω_o the angular revolution frequency; ω_s the angular synchrotron frequency. The bunch power spectrum of the mode *m* is given:

$$h_m(\omega) = \left(\frac{\omega\sigma_z}{c}\right)^{2|m|} \boldsymbol{exp}\left(-\left(\frac{\omega\sigma_z}{c}\right)^2\right) \tag{4}$$

where σ_z is the rms bunch length (for the Gaussian bunch it is assumed that $L = 2\sqrt{\pi}\sigma_z$); c the light velocity. The so-called “chromatic” angular frequency ω_ξ is proportional to the ratio of the chromaticity ξ and the storage ring slippage factor η:

$$\omega_\xi = \omega_0 \frac{\xi}{\eta} \tag{5}$$

The imaginary transverse impedance gives real frequency shifts. By exciting the bunch with transverse kickers (like in the case of tune measurements) and measuring the frequency shift of the betatron lines ($m = 0$) and their sidebands ($m = \pm1; \pm2;...$) as a function of the bunch current we can define the transverse effective impedance and make a suggestion on a suitable transverse broad band impedance model.

The spectrum of the dipole ($m = 0$), quadrupole ($m = \pm1$), sextupole ($m = \pm2$) etc. modes occupy different frequency regions (see eq. (4)). Because of that the coherent modes “probe” different transverse impedances and, as a consequence, have different frequency shifts as a function of current.

At a certain bunch current the mode frequencies can overlap. This leads to the mode instability and the bunch gets lost during a few revolution turns.

Most often the TMCI threshold is defined by the coupling of the dipole (m = 0) and quadrupole (m = -1) modes [4,5]. This is accounted for the fact that, first, the lowest coherent modes have stronger frequency shift as far as it scales inversely with the mode number, $\sim 1/(1+|m|)$. Second, the lowest mode spectrum lie at lower frequencies where, as a rule, the transverse impedance is higher.

Below we report the results of the coherent mode frequency shift measurements in the DAΦNE accumulator ring for the fastest converging modes $m = 0$ and $m = -1$ performed at 200 kV RF voltage and elaborate these data in order to estimate the machine transverse impedance and predict the TMCI threshold.

The measurements were also done at 60 kV, 100 kV and 150 kV RF voltages, but since the results are consistent with those taken at 200 kV and do not give any additional information we omit the data. Here we consider only the vertical plane case as far as the vertical transverse impedance is much higher than the horizontal one because the vertical vacuum chamber size is substantially smaller that the horizontal one. Indeed, during the measurements we did not find any noticeable change of the mode frequencies by varying the bunch current in the range 0 - 100 mA.

Vertical transverse impedance estimate

We used the transverse feedback system kicker in order to excite the bunch and tracked a betatron line and its first sidebands frequencies versus the bunch current. An example of the observed first mode spectrum lines at low current is shown in Fig. 1. One can clearly distinguish the central betatron line and the dipole betatron sidebands (m = +1 and m = -1) on the either side distant by the synchrotron frequency from the central line. The smallest peaks symmetric with respect to the central line correspond to the quadrupole mode betatron sidebands (m = +2 and m = -2).

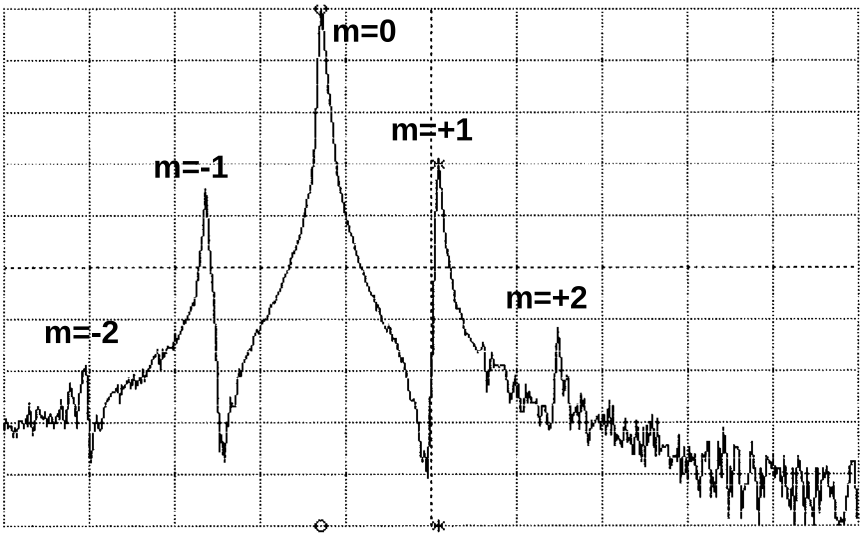

Figure 1: Example of the observed excited betatron line and its first sidebands.

The results of the mode frequency shift measurements at 200 kV RF voltage are presented in Fig. 2.

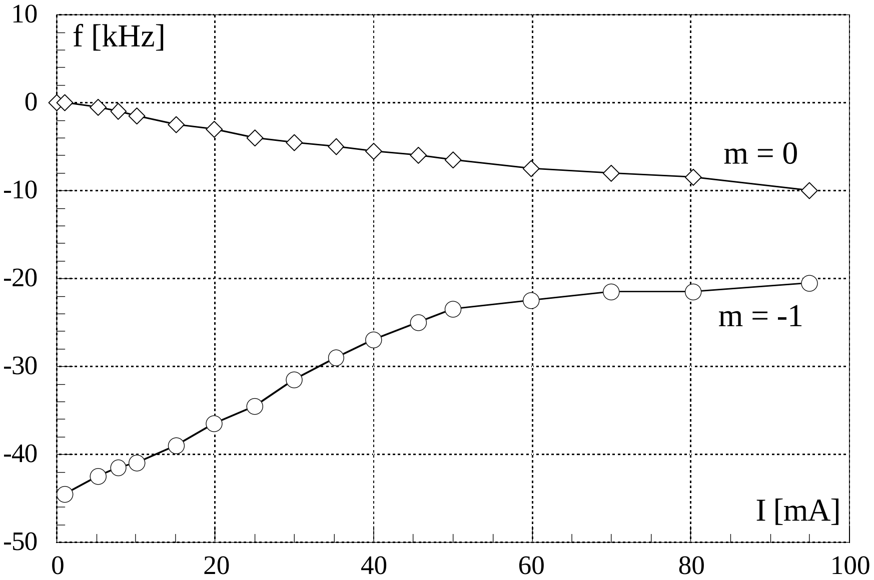

Figure 2: Dipole and quadrupole mode frequency shifts versus bunch current.

However, in order to calculate the effective transverse impedance according to eq. (1) besides the mode frequency shift measurements one has to measure the bunch length as a function of the bunch current. In our case we rely on the numerical simulation results since, as it has been already shown, the bunch lengthening measurements and simulations are in a good agreement for the DAΦNE accumulator ring [6].

The only arising question is how to define the total bunch length L appearing in eq. (1) from the numerical simulations. The bunch distribution differs from Gaussian because due to the interaction with the mostly inductive machine wake field the bunch gets wider acquiring the parabolic shape at high currents. Fortunately, as it is easy to show, the final results does not depend much on by which kind of a distribution we approximate the real bunch distribution. Figure 3 shows the total bunch length as a function of current for two cases: the solid line corresponds to the Gaussian bunch approximation ($L = 2\sqrt{\pi}\sigma_z$) , while the dotted one is based on the fit of the numerical results by the Water Bag distribution. The two data are in an agreement within a few percent accuracy for the whole current range of interest.

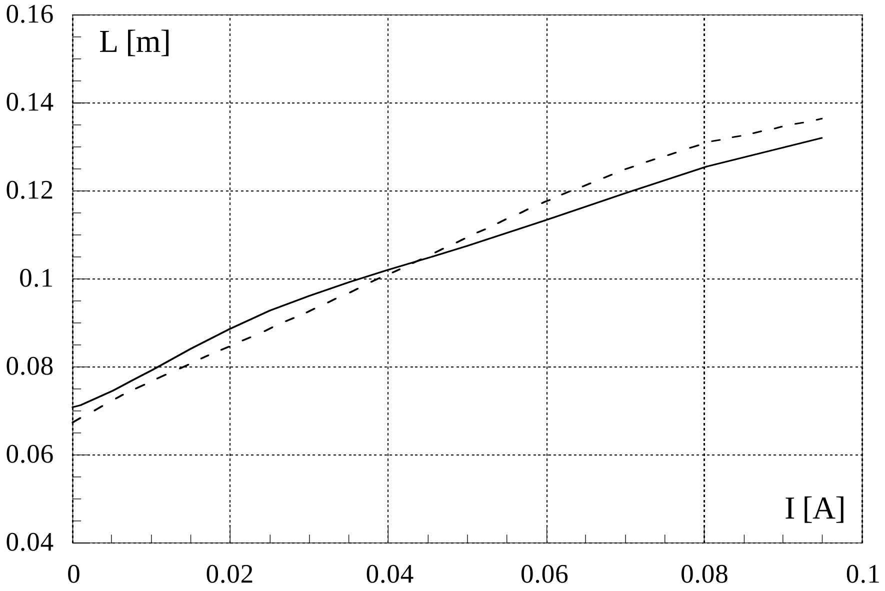

Figure 3: Total bunch length as a function of bunch current.

Finally, the imaginary part of the effective vertical impedance calculated for the dipole mode is shown in Fig. 4.

As it can be seen, the measured effective transverse impedance coincides very well with our earlier prediction of 70 kΩ/m [7], i. e. the transverse impedance of the accumulator ring can be approximated well, at least at low frequencies, by the broad band resonator impedance with the shunt impedance of 70 kΩ/m and the quality factor $Q = 1$.

However, for the bunch currents below 15 mA the effective impedance is lower than this value. Moreover, the lower the current the lower the impedance. We attribute this effect to a strong negative transverse impedance at high frequencies (1.5 - 3 GHz). For lower currents the bunch is shorter and the dipole mode spectrum covers higher frequencies. For a certain bunch length the spectrum start coupling with the negative impedance, thus reducing the effective transverse impedance.

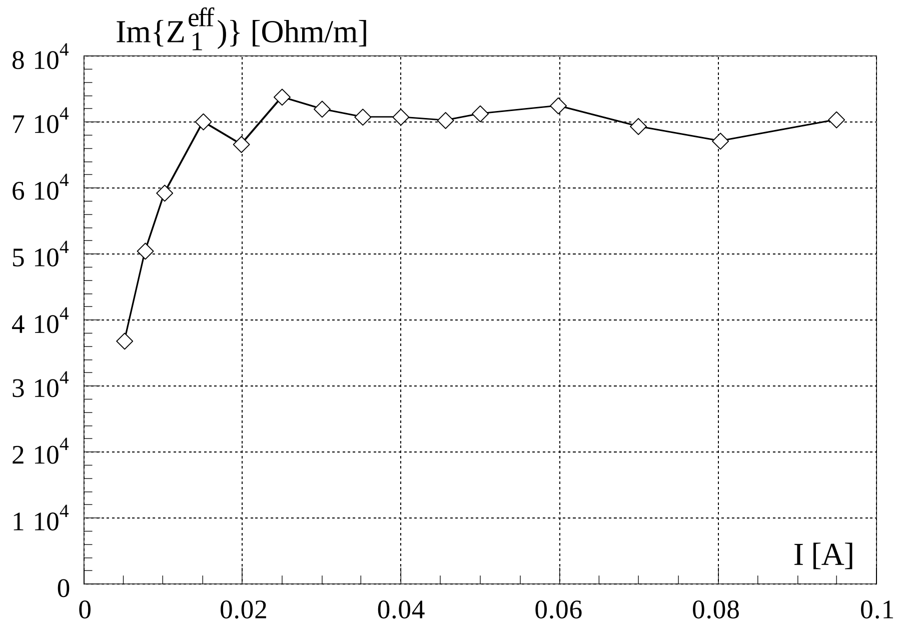

Figure 4: Imaginary part of the transverse effective impedance associated with the dipole mode.

This explanation suits also for the quadrupole mode. In Fig. 2 we observe rather unusual very steep dependence of the mode m = -1 frequency shift on the bunch current. As far as the mode spectrum has a maximum at $\omega \sim c/\sigma_z$, i. e. $f \sim$ 1 - 3 GHz for the accumulator ring, the strong positive shift can be explained by the mode interaction with the strong negative impedance at these frequencies.

In our opinion, the only possible source of such a high negative imaginary impedance in the frequency range is the impedance of parasitic high order modes (HOM) trapped either in the RF cavity or somewhere else in the vacuum chamber discontinuities.

This guess is confirmed by the measurements. Assume the quadrupole mode interacts with a single parasitic HOM. Then, the coherent mode frequency shift can be written as [8]:

$$\Delta\omega_{c_1} = j\frac{Ic}{8\pi Q(E/e)} Z_T\left(\omega_p\right) F\left\{\left(\omega_\xi - (p+Q)\omega_0 - \omega_s\right)\frac{L}{2c}\right\}, \tag{6}$$

where the form-factor F for a bunch with a Water Bag distribution is given by:

$$F(x) = \frac{4}{x^2}\int_0^x J_1(u)u du \tag{7}$$

Here J_1 is the Bessel function of the first order.

Clearly, if a single mode line couples with the HOM (its negative imaginary part) the measured ratio $\Delta\omega_{c_1}/I$ should resemble the shape of the form-factor F. Indeed, as we can see in Fig. 6 the analytical form-factor fits well the measured data in the case when the HOM is situated at $\omega_1/2\pi$ ~2.4 GHz. In this figure the form-factor and measured data are normalized on their maximum values.

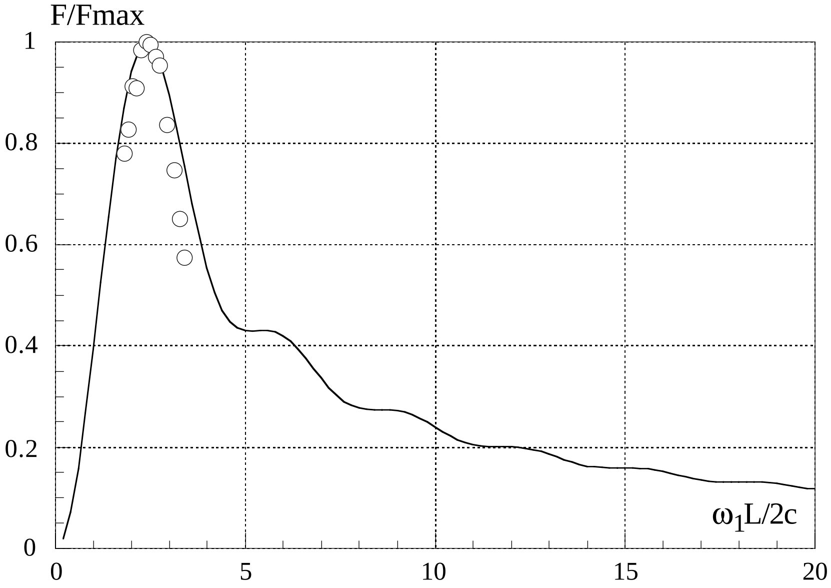

Figure 6: Normalized form-factor as a function of the bunch length: solid line - analytical expression; circles - measured normalized ratio of the frequency shift to the bunch current.

The only small discrepancy is observed for the longer bunches (see the last points in Fig. 6). This is not a surprise since for longer bunches the dipole mode spectrum is shifted towards the lower frequencies where the mode "feels" the positive imaginary part of the broad band impedance.

TMCI threshold

At this point it is quite easy to extrapolate the measured data to the higher currents and evaluate the TMCI threshold.

Again, we find the accumulator bunch length by numerical simulations. Figure 7 shows the bunch length for the bunch currents in the range of 100 - 200 mA.

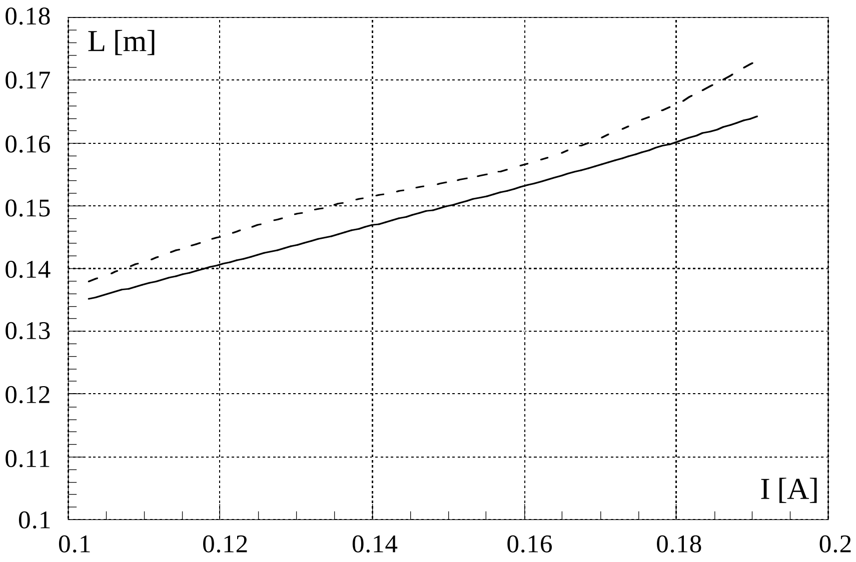

Figure 7: Total bunch length as a function of current: solid line - Gaussian bunch approximation; dotted line - Water bag bunch approximation.

Since by increasing the current the bunch gets longer the dipole mode will interact only with the constant imaginary broad band transverse impedance with $Im\ Z_T = 70\ k\Omega/m$. Therefore, the mode frequency shift in this case will scale as $\sim I/\sigma_z$.

For the quadrupole mode we make a conservative assumption that it is affected only by the found single HOM, i. e a positive contribution of the low frequency impedance is not taken into account. This means that the mode frequency shift is proportional to the bunch current times the form-factor which is calculated analytically knowing the dependence of the bunch length on the current.

The extrapolation of the coherent mode shifts to the higher currents is shown in Fig. 8. As we can see, the modes couple at the current of 160 mA, i. e. the TMCI threshold is by about 20 percent higher than the accumulator ring nominal current of 132 mA. We should again stress here that the value is very pessimistic. However, in order to substantially increase the TMCI threshold, if necessary, one should localize the responsible HOM and try to damped it or to shift its frequency with respect to the coherent mode spectrum lines.

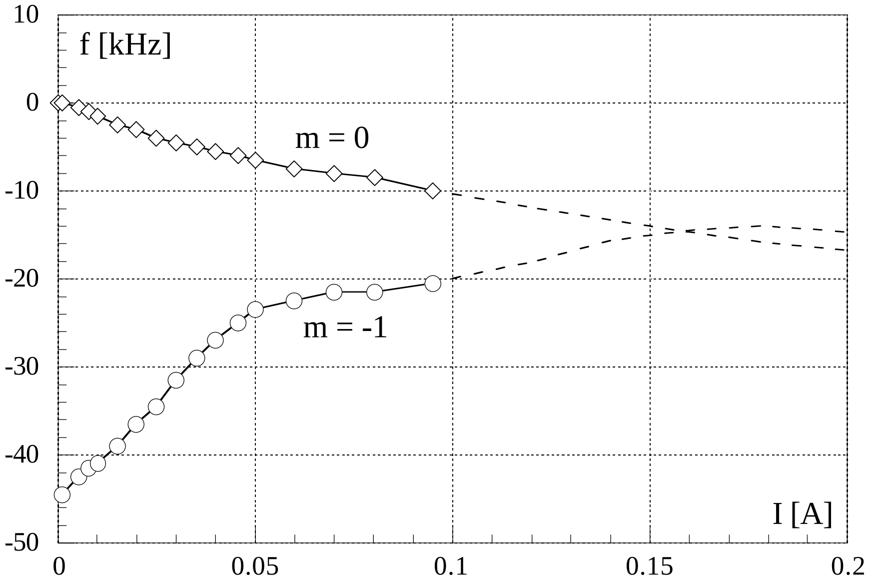

Figure 8: Coupling of the dipole and quadrupole coherent modes.

Conclusions

1) The measurements have shown that the accumulator ring transverse broad band impedance can be approximated by the broad band resonator impedance with the shunt impedance of 70 kΩ/m and the quality factor $Q = 1$. This perfectly agrees with our earlier analytical predictions [7].
2) However, it has been found, that besides the broad band impedance the bunch interacts with a single transverse HOM at f ~ 2.4 GHz.
3) The TMCI is defined by the coupling between the dipole and quadrupole bunch coherent modes. The conservative estimate of the TMCI threshold is 160 mA that is higher than the nominal accumulator ring current.
4) Further TMCI threshold increase can be reached by eliminating or damping the parasitic HOM which strongly shifts the frequency of the bunch quadrupole coherent mode.

References

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