DAΦNE TECHNICAL NOTE

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Note: **G-48**

MEASUREMENT OF TRANSVERSE AND LONGITUDINAL SPECTRA

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OUTLINE

- Longitudinal / Transverse Spectra
  (with / without synchrotron oscillations)
  - Single Particle
    - Many Particles
      - Many Bunches
- Pick-up's, Kickers
- Beam Response
- Beam Transfer Function
  - Cross - BTF
  - BTF with bunched beams

BEAM SPECTRA

References:

- R. Littauer: "Beam Instrumentation", in "Physics of High Energy Particle Accelerators". Editor: M. Month - AIP Conference Proceedings No. 105, pp. 869-953 (1983).
- J.L. Laclare: "Bunched Beam Coherent Instabilities", Cern Accelerator School - Advanced Accelerator Physics Course. Proceedings. Editor: S. Turner - CERN 87-03, p.264 (1987).
- D. Boussard: "Schottky Noise and Beam Transfer Function Diagnostics", Ibid.

Single Particle - Longitudinal

A single particle of charge e rotating with speed v in the central orbit of an accelerator of average radius of curvature R can be described by a time-dependent linear charge density

$$\lambda(t) = \frac{e}{v} \sum_{k=-\infty}^{\infty} \delta(t - kT_0) , \qquad (1.1)$$

where T_O is the revolution time $T_O = 2\pi R/v$ and $\delta(t)$ the impulse function.

By expressing (1.1) as a Fourier series, we write

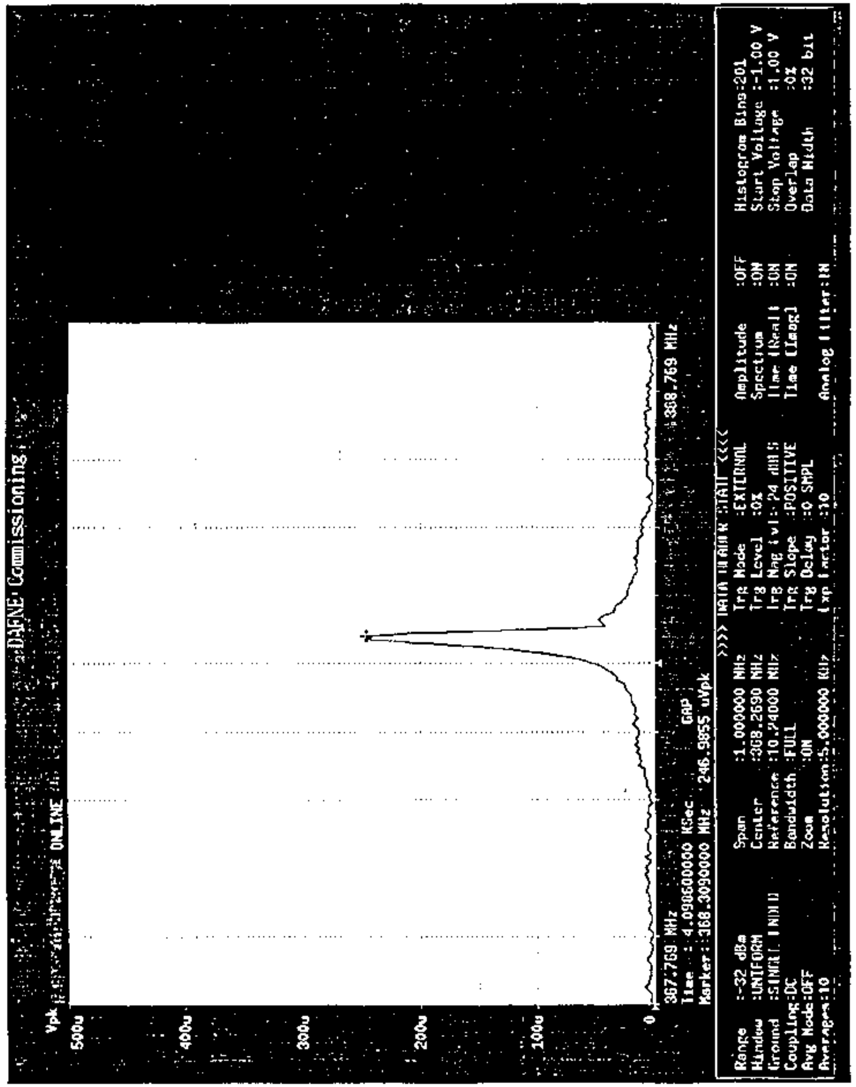
$$\lambda(t) = \frac{e}{vT_0} \sum_{n=-\infty}^{\infty} \exp(jn\omega_0 t) = \frac{e}{2\pi R} \sum_{n=-\infty}^{\infty} \cos(n\omega_0 t) . \qquad (1.2)$$

The frequency spectrum is obtained by the Fourier transform :

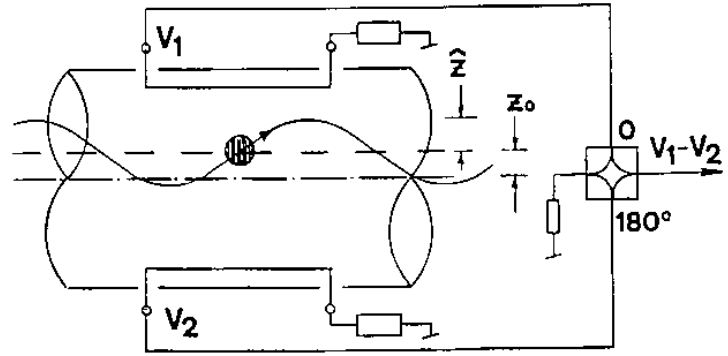
$$\Lambda(\omega) = \frac{e\omega_0}{2\pi v} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) .$$

The line at n=0 is the DC component of the signal, the remaining lines are successive orbital harmonics spaced by w_O. Since $\cos(-n\omega_O t) = \cos(n\omega_O t)$, the negative frequency lines are indistinguishable from those at corresponding positive frequency; the combined amplitude is thus twice the DC component.

A longitudinal pick-up couples to the particle fields, delivering a signal proportional to the linear charge density, whose harmonic content copies that of (1.2) at least up to frequencies of the order $\approx \gamma c/b$, with b the effective radius of the beam pipe and c the speed of light, after which, due to the opening angle of the fields, cut-off occurs.



Single Particle - Transverse



A suitable configuration of pick-up's forms a beam position monitor (BPM), used to measure the horizontal or vertical transverse displacement from the design orbit. We use z to indicate the generic transverse position. A BPM is usually sensitive also to the current intensity so that the measured quantity is actually proportional to the *linear dipole density d*, defined as the product of the linear charge density λ times the position z.

$$d = \lambda \cdot z .$$

Let's write the position z as the superposition of two terms

$$z(t) = z_0 + \hat{z} \cos(\omega_\beta t) , \tag{1.3}$$

where z_0 is a stable offset due, for example, to a closed orbit distortion or to a BPM misalignment, or both, and the second term is the oscillatory one, due to the betatron oscillation, with $\omega_\beta = Q\omega_0$ the betatron angular frequency and Q the betatron tune.

The resulting linear dipole density is then obtained by multiplying (1.2) by (1.3)

$$d = z_0 \frac{e}{2\pi R} \sum_{n=-\infty}^{\infty} \cos(n\omega_0 t) + \hat{z} \frac{e}{2\pi R} \sum_{n=-\infty}^{\infty} \cos(n\omega_0 t) \cos(\omega_\beta t) . \qquad (1.4)$$

The first term gives terms similar to (1.2) in the frequency content, but weighted by the closed orbit. The second term has a different signature and contains information on the betatron motion. If this latter is of interest, the closed term is rejected by electronic means, or by centering the beam or even by centering the BPM itself.

By considering only the second term in, the linear dipole density may be written as

$$d = \hat{z} \frac{e}{2\pi R} \sum_{n=-\infty}^{\infty} \cos((n + Q)\omega_0 t) , \qquad (1.5)$$

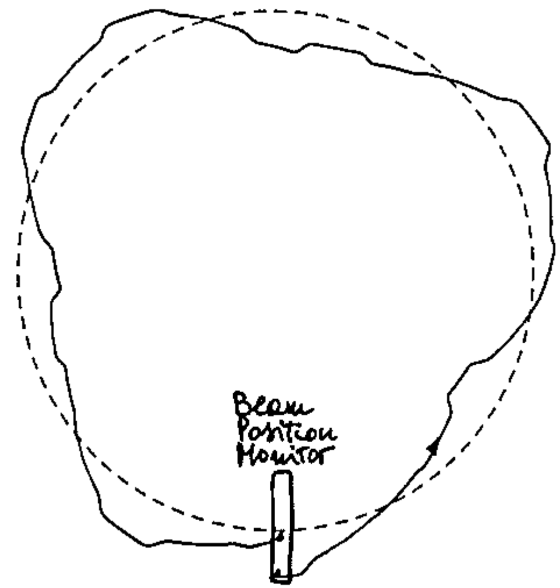
showing the appearance of a whole set of side-bands beside the harmonics of the revolution frequency, produced by the non-linear operation of sampling the betatron motion at finite intervals of time.

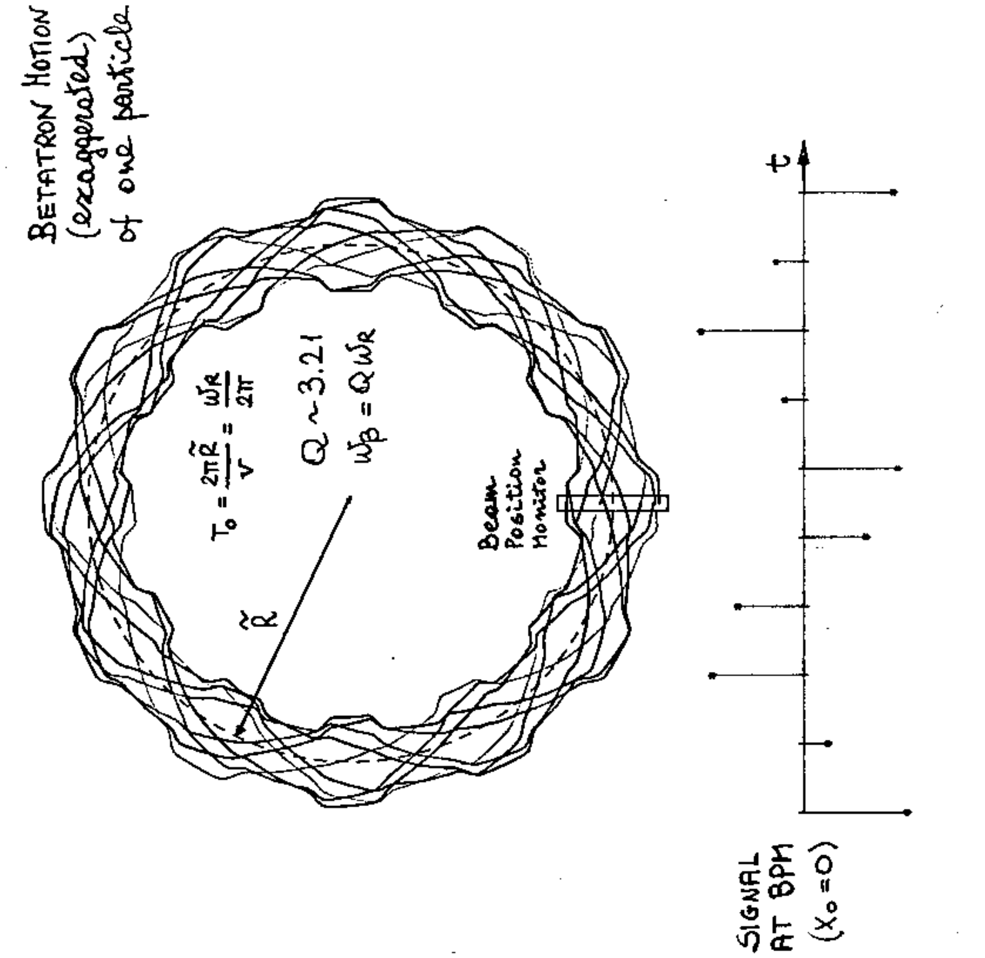
It is interesting to express (1.5) in terms of positive frequencies only, as seen with a conventional spectrum analyzer. To this purpose we first write $Q = M + q$, with M the integer part and q the fractional part of Q, and obtain

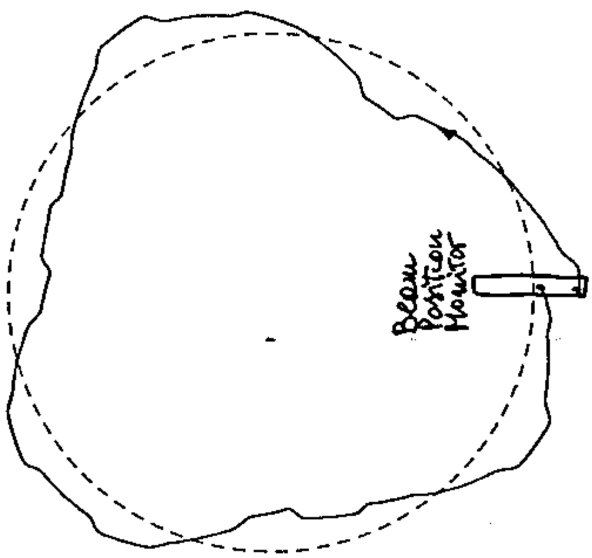
$$d = \hat{z} \frac{e}{2\pi R} \left\{ \cos(q\omega_0 t) + \sum_{n'=1}^{\infty} \cos\left[(n' \pm q)\omega_0 t\right] \right\} , \qquad (1.6)$$

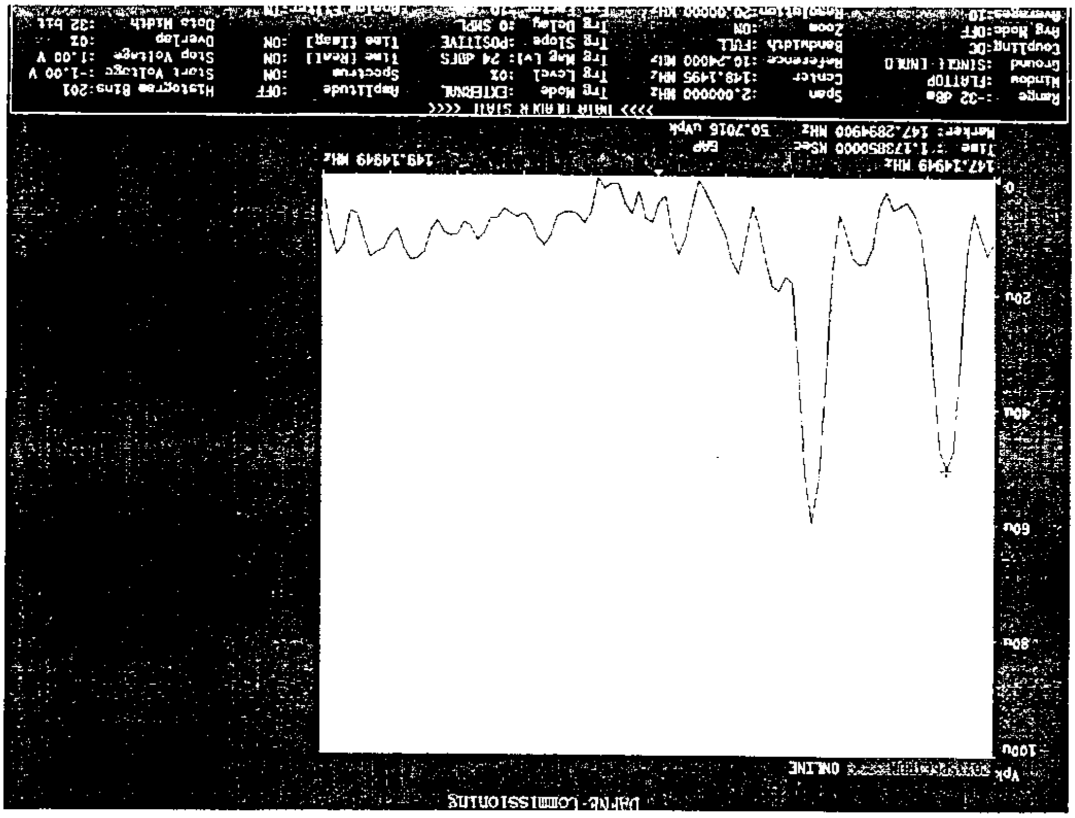
where the new index $n' = n + M$ has been introduced.

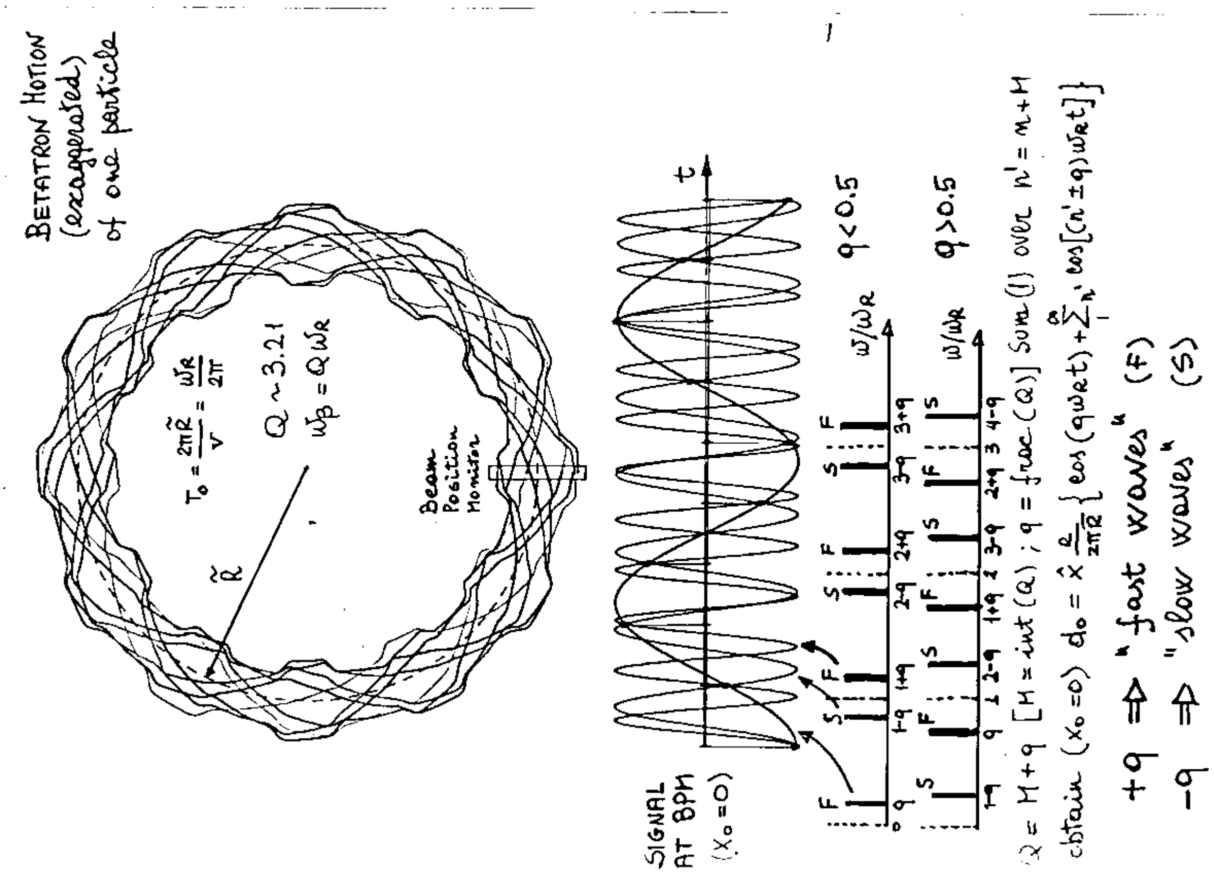
The components of the spectrum (1.6) with $+q$ are called *fast waves*. Those with $-q$ are called *slow waves*. If the value of q is less than 1/2 ("above the integer") the fast waves stand at the high-frequency sides of the revolution harmonics and the slow waves at the low-frequency sides; when q is greater than 1/2 ("below the integer") the opposite relationship holds.




BETATRON MOTION
(exaggerated)
of one particle
$T_0 = \frac{2\pi\tilde{R}}{v} = \frac{\omega_R}{2\pi}$
$Q \sim 3.21$
$\omega_\beta = Q\,\omega_R$
$\tilde{R}$
Beam
Position
Monitor
t
SIGNAL
AT BPM
$(X_0 = 0)$


Beam
Position
Monitor




BETATRON MOTION (exaggerated) of one particle
Beam Position Monitor
SIGNAL AT BPM
q<0.5
q>0.5
"fast waves" (F)
"slow waves" (S)

Longitudinal Spectra with Synchrotron Satellites

In the presence of an RF accelerating field, a particle beam is bunched; the single particle undergoes synchrotron oscillations of the instantaneous energy.

The angular frequency of revolution is affected according to

$$\frac{d\omega}{\omega_0} = -\eta \frac{dp}{p_0}, \tag{1.7}$$

where $d\omega$ is the frequency variation, dp is the instantaneous momentum deviation with respect to the nominal value p_0 and η is defined by

$$\eta = \left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right), \tag{1.8}$$

where γ_t is the transition energy at which the increase of velocity corresponding to a momentum increase is compensated by the increase of orbit length, thus leaving the revolution time unaltered. The term $(1/\gamma_t)^2$ is called the *momentum compaction factor* α_c. At ultra-relativistic energies the second term in (1.8) becomes negligible and (1.7) is written

$$\frac{d\omega}{\omega_0} \approx -\alpha_c \frac{dp}{p_0}.$$

The time between successive passages measured at the monitor is

$$T_0 + \tau = T_0\left[1 + \frac{\tau_s}{T_0}\cos(\Omega_s t + \psi)\right], \tag{1.9}$$

where Ω_s is the angular frequency of the synchrotron oscillation, ψ is a phase constant, τ_s is the amplitude of time-modulation and

$$\left(\frac{\Delta\tau}{T_0}\right) = -\left(\frac{d\omega}{\omega_0}\right) = \frac{d\tau}{dt}. \tag{1.10}$$

The linear charge density is

$$\lambda(t) = \frac{e}{V}\sum_{k=-\infty}^{\infty} \delta(t - kT_0 - \tau).$$

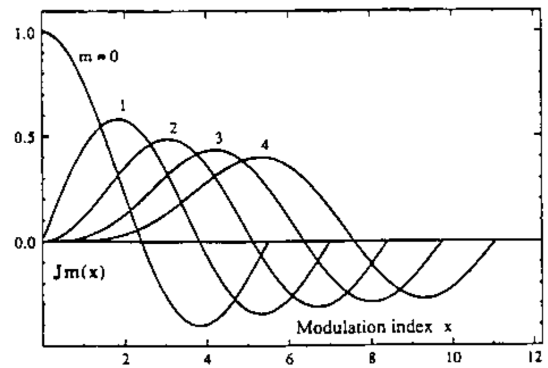
Using $\exp[jx\cos(y)] = \sum_{m=-\infty}^{\infty} (j)^m J_m(x)\exp(jmx)$

and (1.2), we can express the linear charge density as a Fourier series

$$\lambda(t) = \frac{e}{2\pi R}\sum_{n,m=-\infty}^{\infty} (-j)^m J_m(n\omega_0\tau_s)\exp[j(n\omega_0 + m\Omega_s)t + m\psi]. \tag{1.11}$$

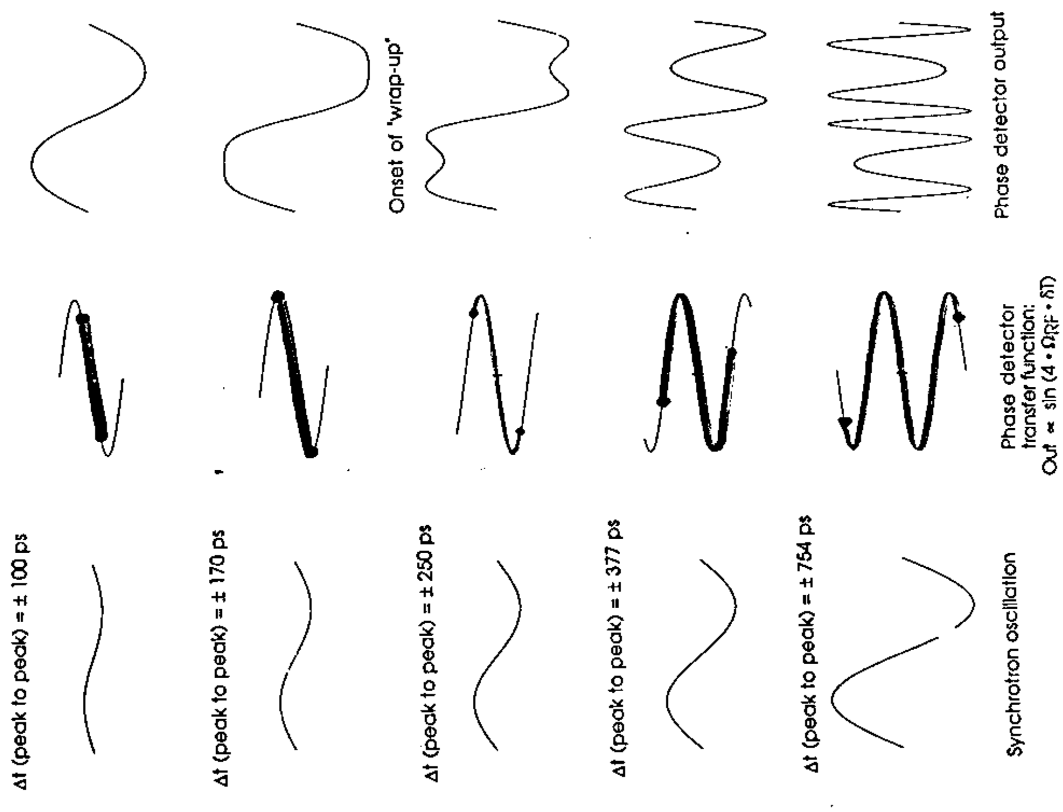
Each original line in the spectrum (1.2) has now degenerated into an infinite set of satellites right and left at $\pm\Omega_s, \pm2\Omega_s, \ldots, \pm m\Omega_s$ with the amplitudes modulated by the Bessel functions of the first kind of order m, J_m.

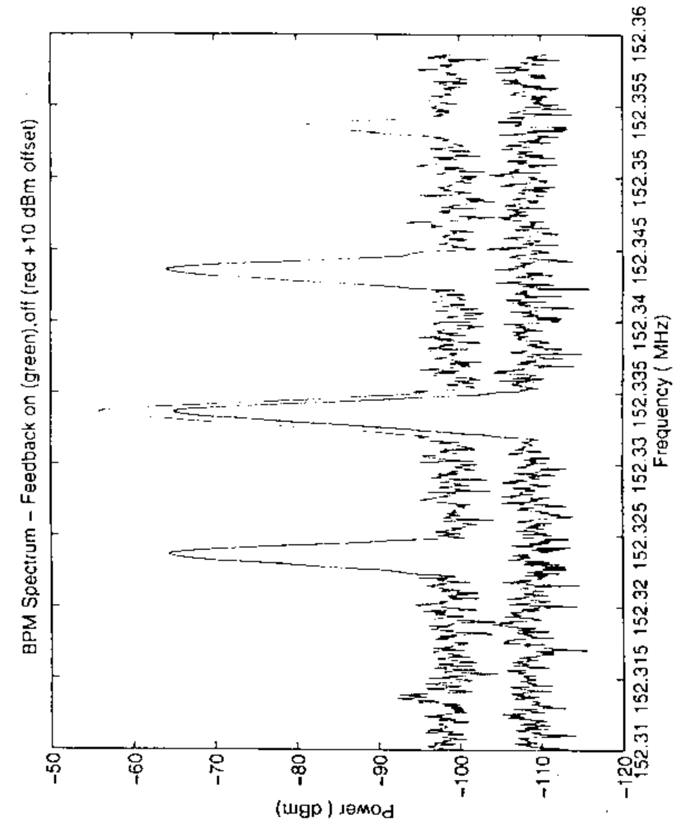
The argument $n\omega_0\tau_s$ corresponds to the phase-modulation index used in telecommunications. Although there is a nominally infinite number of side-bands, only a finite number are of appreciable amplitude: namely the coefficients $J_m(n\omega_0\tau_s)$ fall-off very rapidly beyond $m \sim n\omega_0\tau_s$.

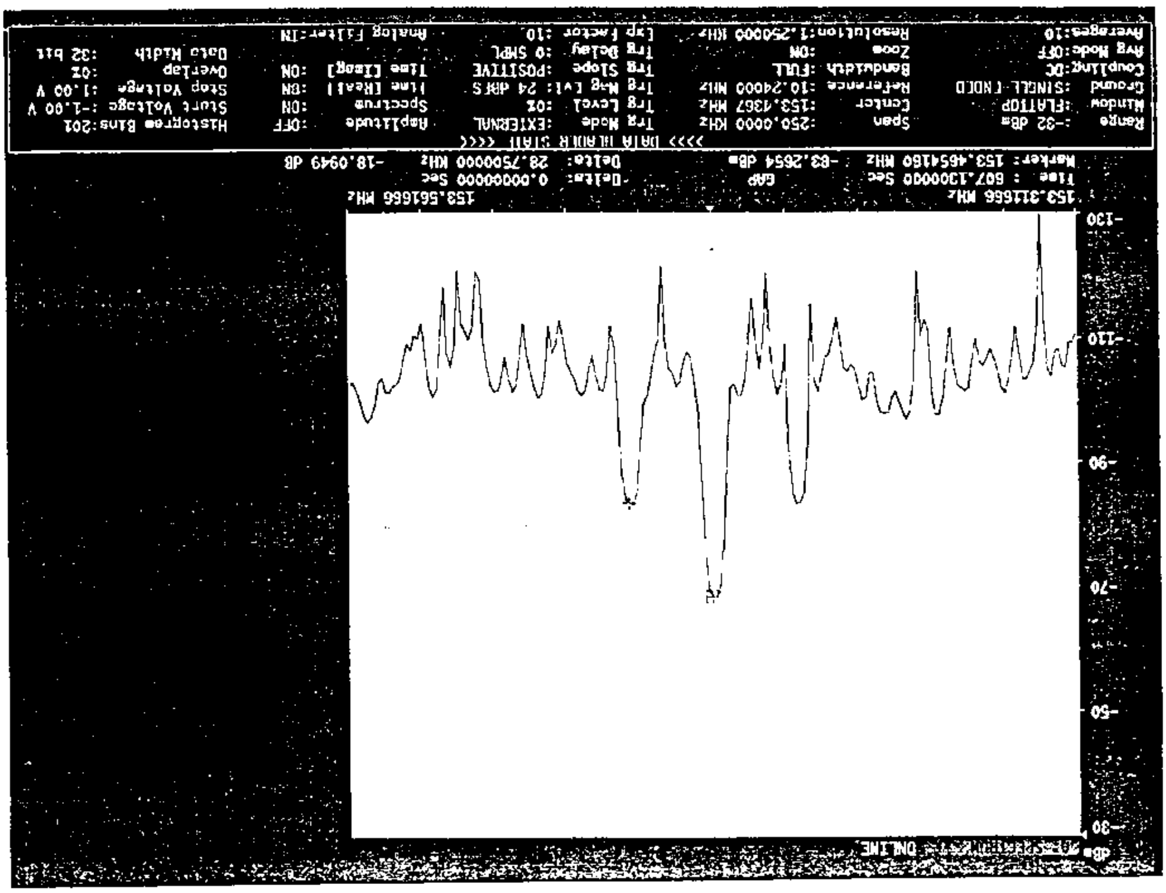


Qualitative sketch of the behavior of the Bessel function Jm(x).

PHASE DETECTOR "WRAP-UP"
(included in the simulation program)




BPM Spectrum – Feedback on (green),off (red +10 dBm offset)
Power (dBm)
Frequency (MHz)



Transverse Spectra with Synchrotron Satellites

We must take into account the modulation of the betatron tune due to the energy modulation. In fact, particles with energy deviating from the nominal value are focused differently. The chromaticity of a machine is the relative change of tune of a particle with relative momentum deviation dp

$$\xi = \text{chromaticity} = \frac{dQ}{dp} \cdot \frac{P_0}{Q_0} , \qquad (1.12)$$

where Q_o is the tune value pertaining to the nominal energy.

The rate of change of the betatron phase μ_β in the presence of energy oscillations is then, to first order,

$$\dot{\mu}_\beta = \omega_\beta \approx \omega_0 Q_0 \left(1 + \frac{d\omega}{\omega_0} + \frac{dQ}{Q_0}\right) = \omega_0 Q_0 \left[1 - \frac{\Delta\tau}{T_0}\left(1 - \frac{\xi}{\eta}\right)\right] \qquad (1.13)$$

and, taking into account the time dependence (1.9) of the time of passage and its rate of change (1.10), the betatron phase is

$$\mu_\beta(t) = \omega_0 Q_0 t + (\omega_\xi - \omega_0 Q_0)\tau_s \cos(\Omega_s t + \psi) , \qquad (1.14)$$

where the chromatic frequency $\omega_\xi = (\xi Q_o/\eta)\omega_o$ has been introduced.

The expression of the linear dipole density d is now written, taking into account (1.11) and (1.14), as

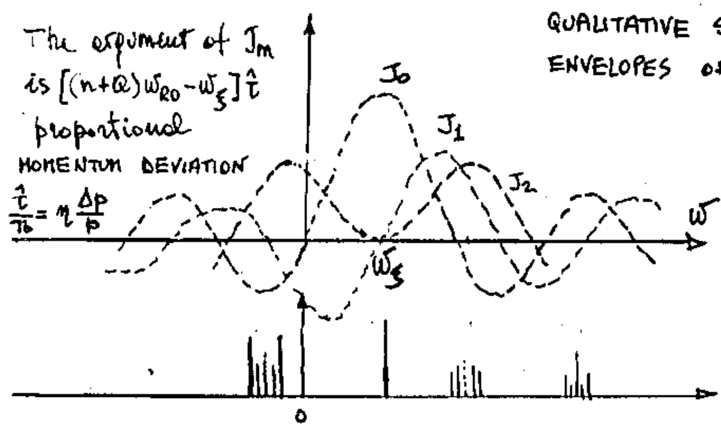
$$d(t) = \frac{e\hat{z}}{2\pi R} \sum_{n,m=-\infty}^{\infty} (-j)^m J_m\left\{\left[(n+Q)\omega_0 - \omega_\xi\right]\tau_s\right\} \exp\left[j(\omega_{nm} t + m\psi)\right] \quad (1.15)$$

with the *mode frequency* $\omega_{nm} = (n + Q)\omega_0 + m\,\Omega_s$.

Due to the tune modulation (1.13), the amplitude envelope function is shifted by the chromatic frequency wx. Thus, examining with a spectrum analyzer (positive frequencies only) the slow and fast waves straddling a harmonic of the revolution frequency, the mode amplitudes above and below may be quite different due to the different argument of the modulating Bessel function. Namely, across the high and low frequency sides of the n-th revolution harmonic the satellites amplitude is modulated by

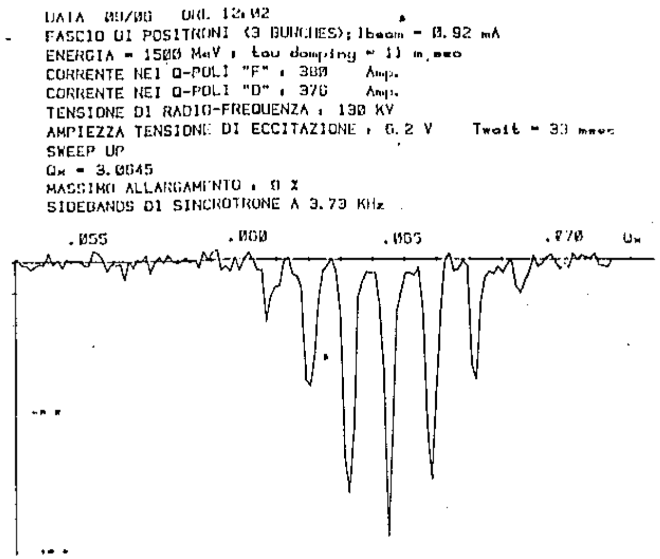
$$\text{Fast waves} \rightarrow \left| J_m\left\{\left[(n+q)\,\omega_0 - \omega_\xi\right]\tau_s\right\}\right|$$

$$\text{Slow waves} \rightarrow \left| J_m\left\{\left[(n-q)\,\omega_0 + \omega_\xi\right]\tau_s\right\}\right| .$$



QUALITATIVE SKETCH OF ENVELOPES OF (1.15)

IF WE KICK THE PARTICLE WITH A LATERAL FORCE AT A FREQUENCY WHERE ANY BETATRON LINE OR SATELLITE IS PRESENT, WE OBSERVE A RESONANT RESPONSE



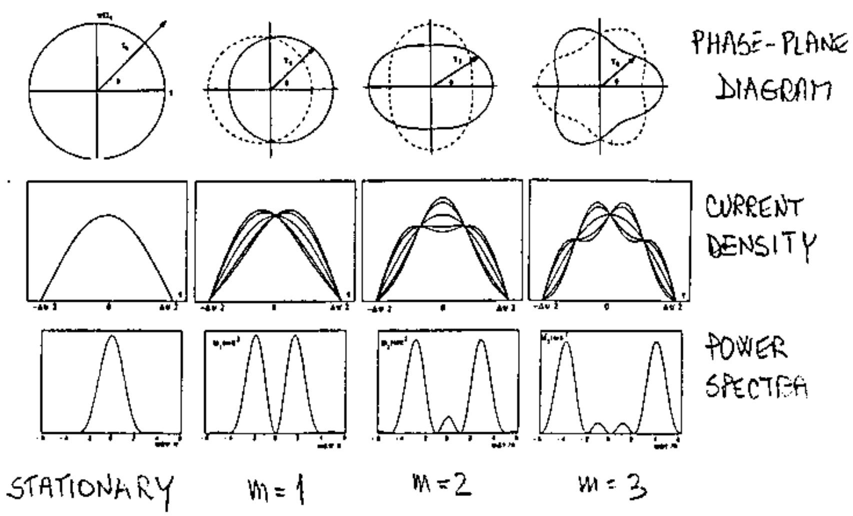
EXAMPLE OF MEASURED SPECTRUM EXHIBITING SYNCHROTRON SATELLITES

Single Bunch

Longitudinal coherent modes. The actual beam can be considered as a beam with a stationary distribution $g_0(\tau_s)$ in the longitudinal phase-space, plus some small density modulation Σg_m , which always exists due, for example, to previous beam manipulations such as injection and bunching for protons, and in general, to the interaction with the machine impedances:

$$g_m(\tau_s, \phi) = R_m(\tau_s) e^{jm\phi} \, . \qquad (1.16)$$

Each pattern g_m rotates in the longitudinal phase space at a frequency $m\Omega_s + \Delta\omega_{lm}$, where m=1 for dipole modes, m=2 for quadrupole modes, m=3 for sextupole modes, etc. $\Delta\omega_{lm}$, is a coherent frequency shift, depending, for example, on bunch current, machine impedance, feedback system and bunch length.



The coherent modes show a line spectrum at frequencies

$$\omega = n\omega_0 + m\Omega_s + \Delta\omega_{lm} \qquad (-\infty \le n, m \le \infty)$$

where $\Delta\omega_{lm}$ is a coherent frequency shift (in the single particle spectrum only lines $n\omega_0 + m\Omega_s$ contribute to the spectrum of the m-th coherent pattern).

The m-th mode corresponds to m+1 half wavelengths of a line density modulation along the bunch. The spectrum of such a perturbation has a broad maximum at $\omega_m \sim (m+1)\pi/\Delta\tau$ and extends over a frequency range of $\Delta\omega \sim \pm 2\pi/\Delta\tau$.

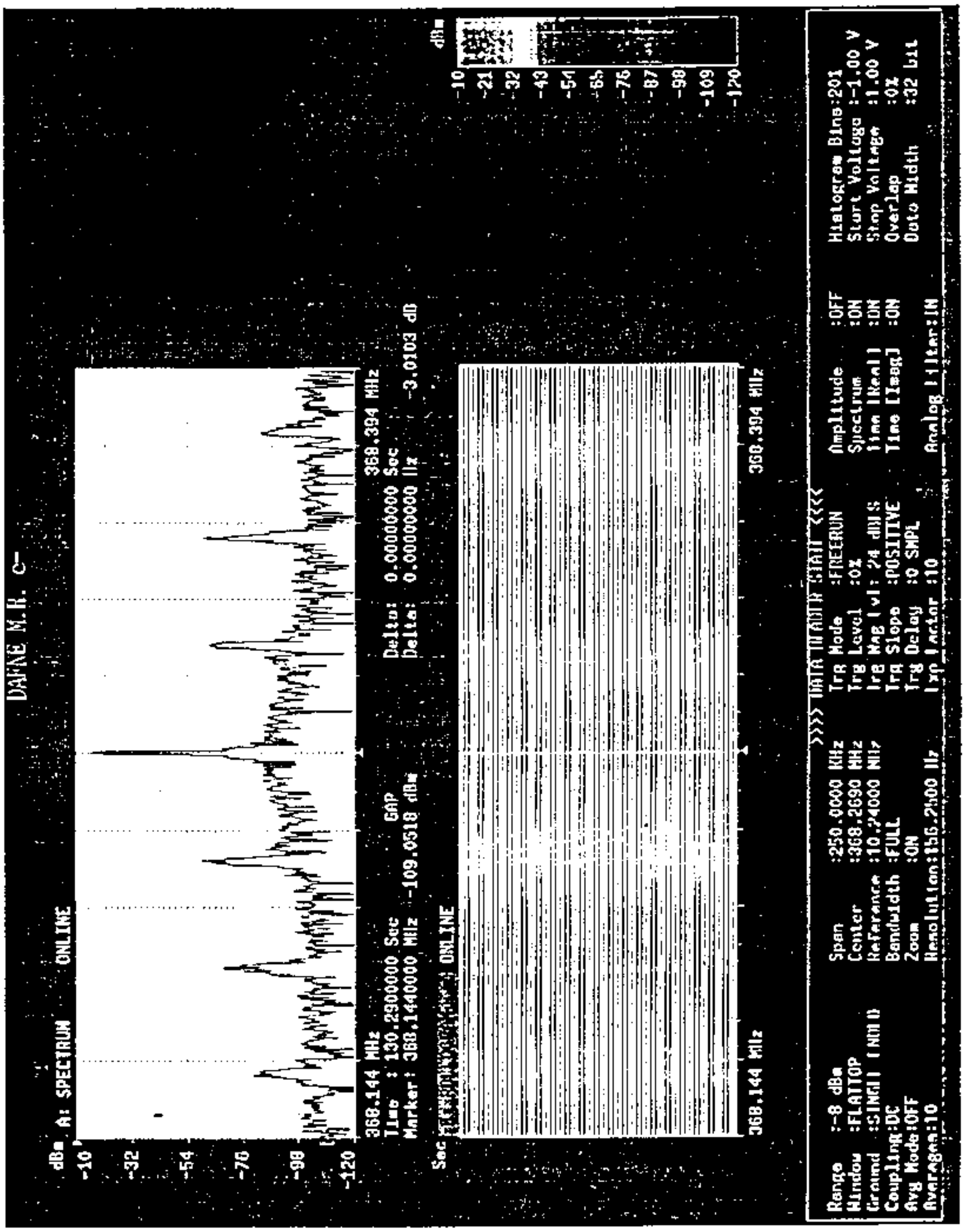
Also for the transverse case we have a line spectrum at angular frequencies

$$\omega = (n + q)\omega_0 + m\Omega_s + \Delta\omega_{tm} \qquad (-\infty \le n, m \le \infty) \, .$$

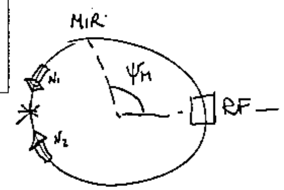
where $\Delta\omega_{tm}$ is a coherent frequency shift.

Some differences in the spectrum of the transverse signal should be mentioned with respect to the longitudinal one:

- the transverse signal induced by a stationary distribution is null;
- a coherent transverse mode m=0 is present, corresponding to a dipolar transverse oscillation of the center of mass of a bunch with a stationary distribution in the longitudinal phase space;
- the spectrum amplitude is peaked at ω_ξ for mode m=0 and at $\sim\omega_\xi \pm (m+1)\,\pi/\Delta\tau$ for other modes.



TWO BEAMS



$\tilde{R} = 16.716$

$$i_1 = N_1 e \sum_{-\infty}^{\infty} \delta(t - t_n - kT_0),$$

$$t_n = \psi_n / \omega_0 \; ; \; \omega_0 = c/\tilde{R} \; ; \; d_n = \psi_n R \; , \; d_n = \frac{2\pi\tilde{R}}{4} + 0.4$$

$$\psi_n = \left[\frac{2\pi\tilde{R}}{4} + 0.4\right] / \tilde{R} = \frac{\pi}{2} + \frac{0.4}{\tilde{R}} = 1.595 \text{ rad}$$

$$i_2 = N_2 e \sum_{-\infty}^{\infty} \delta(t - (T_0 - t_n) - kT_0) = N_2 e \sum \delta(t + t_n - kT_0)$$

$$i_1 = \frac{N_1 e}{T_0} \sum_{-\infty}^{\infty} e^{jn\omega_0(t - t_n)}$$

$$i_2 = \frac{N_2 e}{T_0} \sum_{-\infty}^{\infty} e^{jn\omega_0(t + t_n)}$$

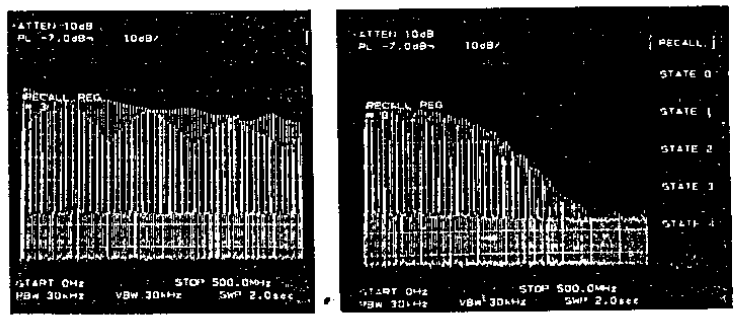
$$i_1 = \frac{N_1 e}{T_0} \sum_{-\infty}^{\infty} e^{jn(\omega_0 t - \psi_n)}$$

$$i_2 = \frac{N_2 e}{T_0} \sum_{-\infty}^{\infty} e^{jn(\omega_0 t + \psi_n)}$$

$$i_1 + i_2 = \frac{e}{T_0} \sum_{-\infty}^{\infty} N_1 e^{jn(\omega_0 t - \psi_n)} + N_2 e^{jn(\omega_0 t + \psi_n)}$$

$$(i_1 + i_2)_n = \frac{2e}{T_0} \left\{ (N_1 + N_2) \cos n\omega_0 t \cos n\psi_n + (N_1 - N_2) \sin n\omega_0 t \sin n\psi_n \right\}$$

$$\cos n\psi_n = C_n$$
$$\sin n\psi_n = S_n$$



$$I_n = [\langle i_+ \rangle f_i(x_+, y_+) + \langle i_- \rangle f_i(x_-, y_-)] \cos(n\vartheta) + \\ + j[\langle i_- \rangle f_i(x_-, y_-) - \langle i_+ \rangle f_i(x_+, y_+)] \sin(n\vartheta)$$

A beam, consisting of M similar and equally-spaced bunches can oscillate coherently in M distinct modes, depending on the phase relationship between the individual oscillations. The linear charge density is a sum of M contributions (single particles implied !):

$$\lambda(t) = \frac{e}{V} \sum_{b=1}^{M} \sum_{k=-\infty}^{\infty} \delta \left(t - \left(\frac{b}{M} + k\right) T_0 - \tau_b\right) , \qquad (1.17)$$

where $\tau_b = \tau_s \cos\left(\Omega_s t + \psi_b\right)$.

Proceeding in the same way as in (1.2) and (1.11), we get :

$$\lambda(t) = \frac{e}{2\pi R} \sum_{n,m=-\infty}^{\infty} (-j)^m J_m(n\omega_0 \tau_s) \exp\left[j\left(n\omega_0 + m\Omega_s\right)t\right] \cdot \sum_{b=1}^{M} \exp\left[j\left(m\psi_b - \frac{2bn\pi}{M}\right)\right] \qquad (1.18)$$

The last Σ is equal to M , provided that the phase shifts between the perturbations of two adjacent bunches satisfy

$$m(\psi_{b+1} - \psi_b) = \frac{2p\pi}{M} , modulo 2\pi , \qquad (1.19)$$

where p can be 0, 1, 2, .., M-1, defining the p-th mode of coherent coupled bunch motion. Otherwise the last Σ is zero.

So, for M similar bunches, M distinct *longitudinal* coherent coupled bunch modes can be excited. The spectrum of the p-th mode is at frequencies:

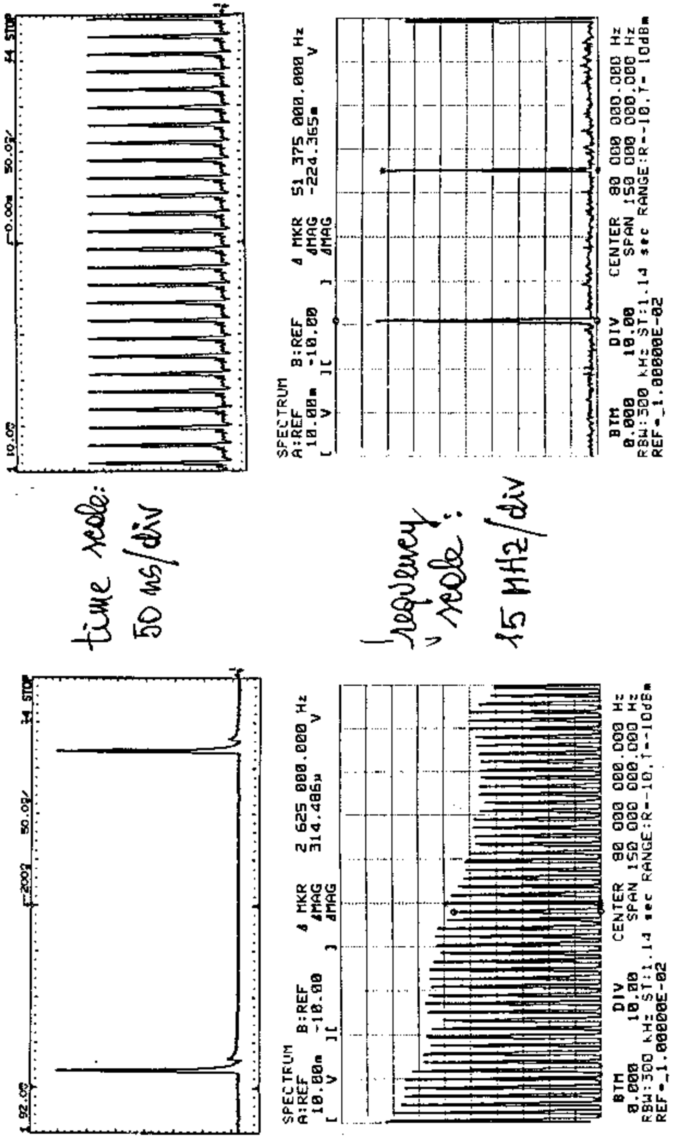
$$\omega_k = (kM + p)\omega_0 + m\Omega_s ,$$

with k running from $-\infty$ to $+\infty$. The amplitude of the spectrum lines is M times larger than in (1.11), but M times more spaced in frequency.

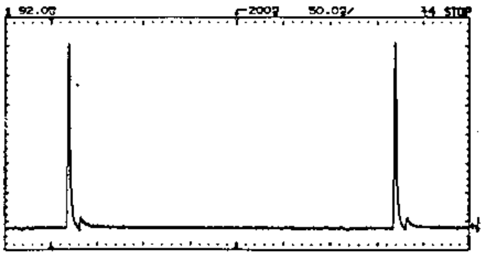
It can be shown also for the *transverse* motion that for M equally spaced bunches, only every M-th line occurs for every p-th coupled mode:

$$\omega_k = (kM + p + Q)\omega_0 + m\Omega_s .$$

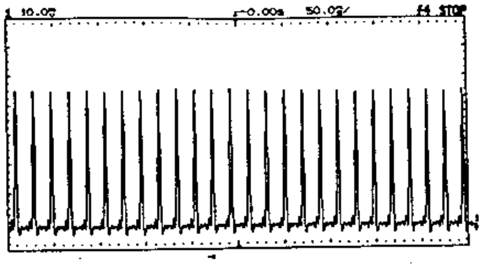
ADONE : Revolution time 350.12 ns (frev = 2.856 MHz)
RF harmonic number = 18

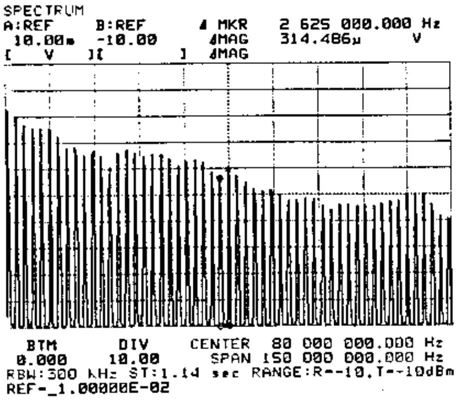


ADONE: Revolution time 350.12 ns (f_{REV} = 2.856 MHz)
RF harmonic number = 18

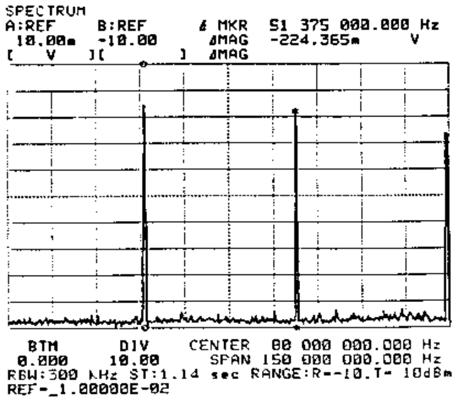


time scale: 50 ns/div





frequency scale: 15 MHz/div

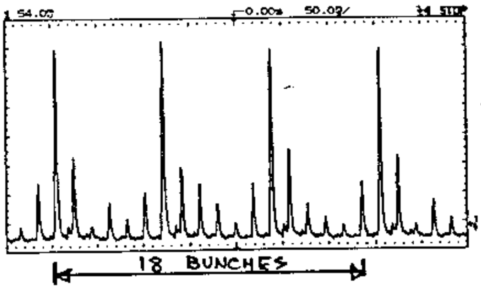


1 BUNCH

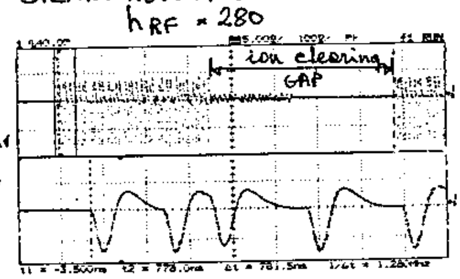
18 BUNCHES

If the bunches carry unequal charges or they are not equally spaced, we see a more complicated pattern of frequency lines

SPEAR: Revolution time: 781 ns
h_{RF} = 280

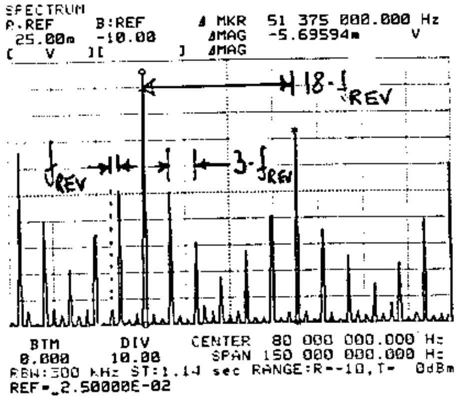


50 ns/div



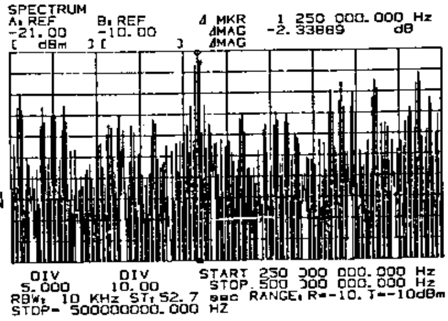
100 ns/div

5 ns/div



15 MHz/div

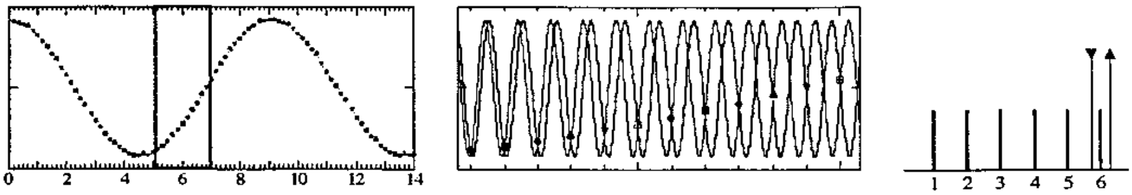
25 MHz/div



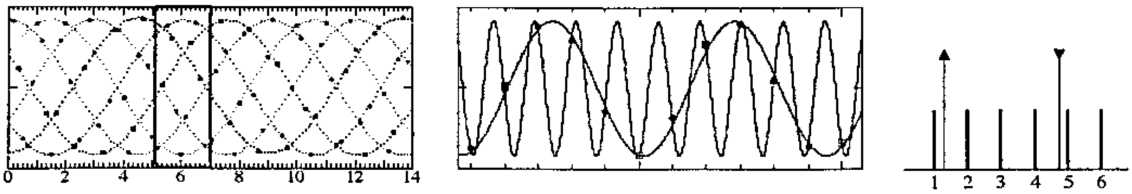
ADONE.

SPEAR

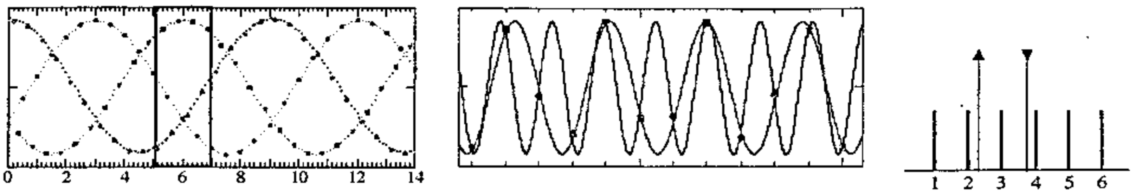
Mode 0



Mode 1



Mode 2



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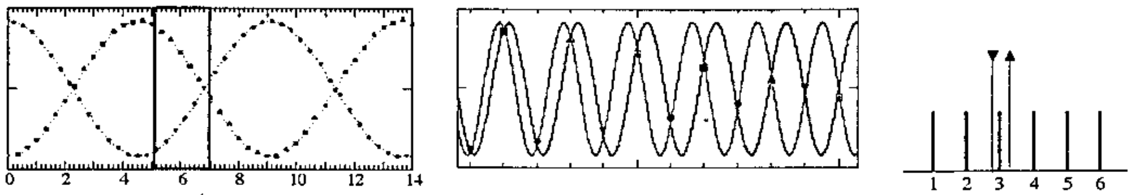

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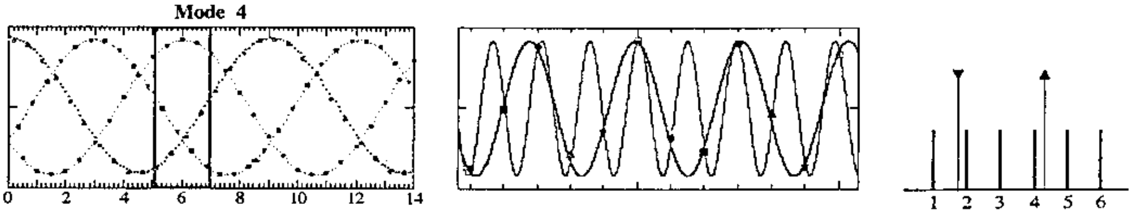
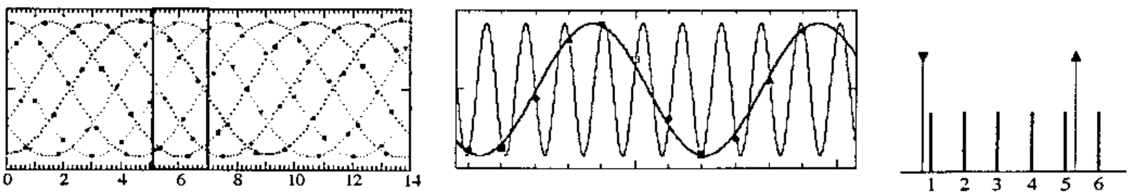


Mode 5



MULTIBUNCH MODES

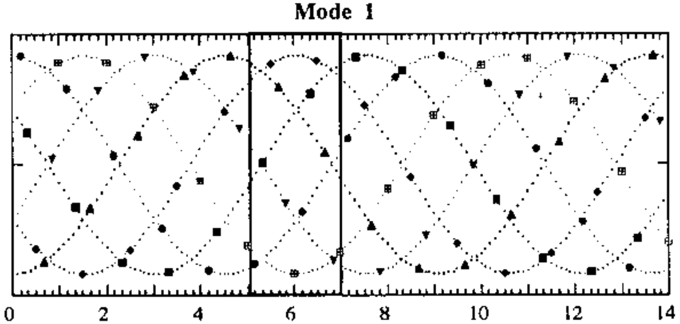
Non-zero terms at angular frequencies

$$\omega_{p,n} = (pM + n + mQ_s)\omega_0$$

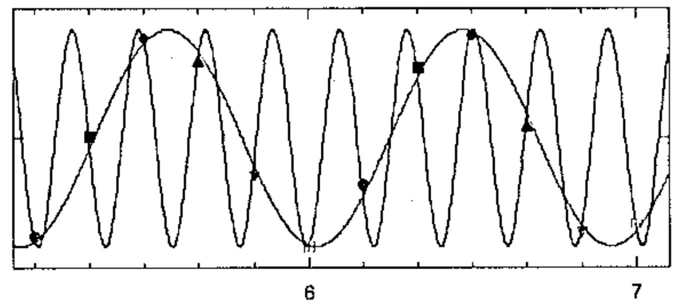
Qs, synchrotron tune,
$-\infty < p < +\infty$,
$n = 0, 1, 2, .. M-1$, (n-th mode of coherent CB motion),
$m = 1, 2, 3, \dots$, (phase-plane periodicity).

The phase shifts between the perturbations of two adjacent bunches satisfy

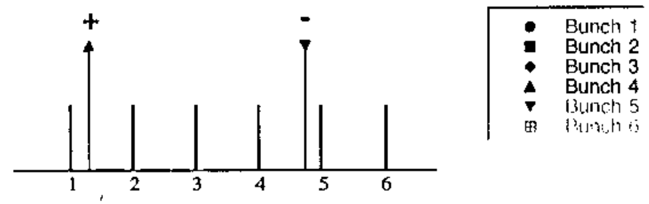
$$(\psi_{b+1} - \psi_b) = \frac{2n\pi}{M}$$



Detected coherent oscillation of $M = 6$ bunches with phase advance $2\pi/6$ ($n = 1$)



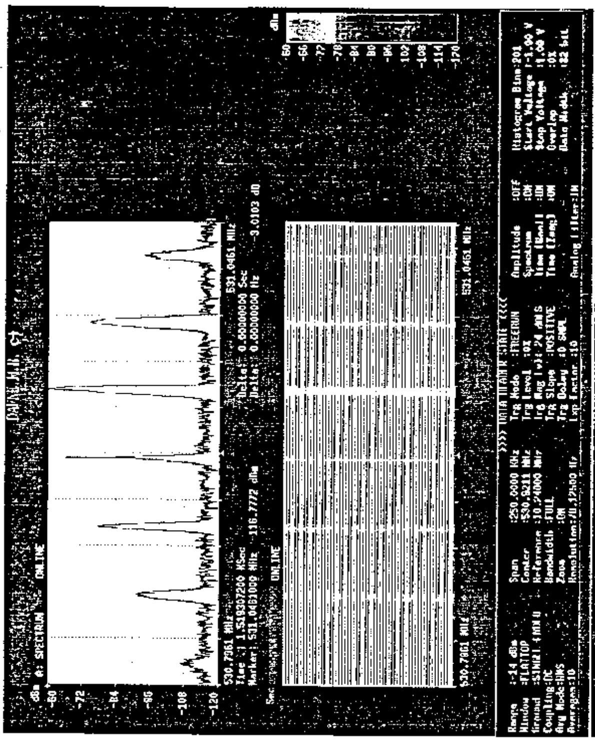
Enlarged view with superimposed two high frequency modes at $(1+Q_s)\omega_0$ and $(-6+1+Q_s)\omega_0$.

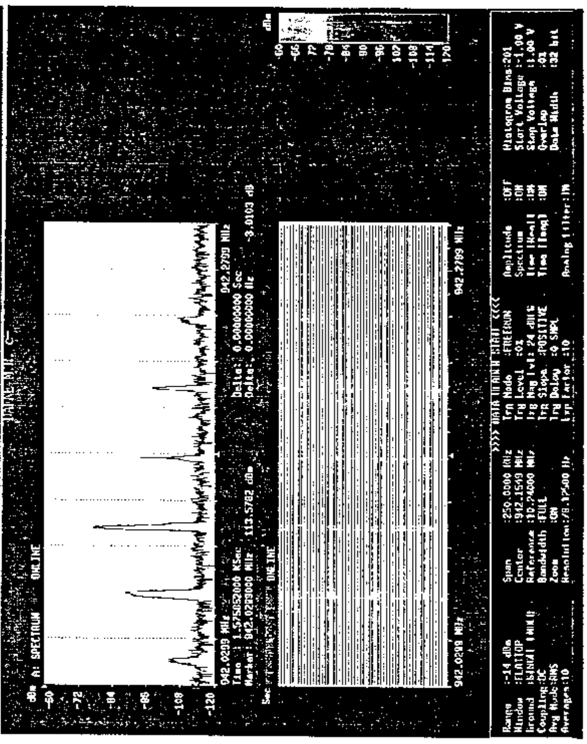


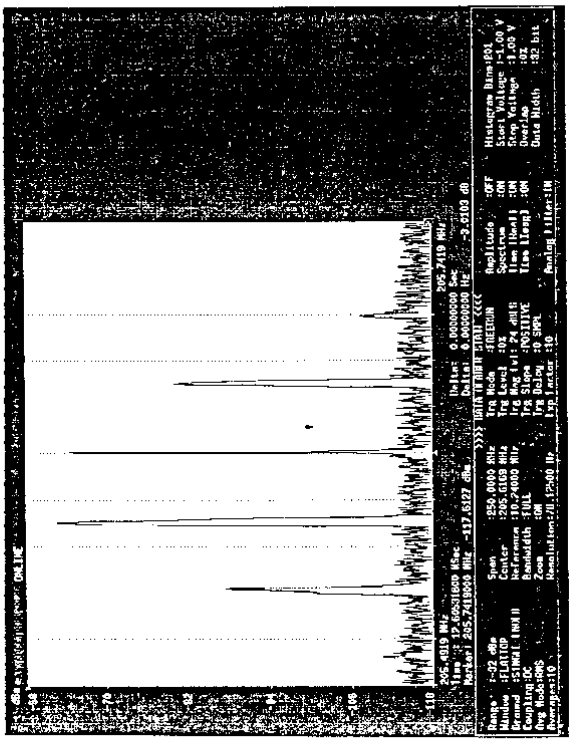
Spectrum analyzer representation (positive frequencies only) of modes above. The pattern repeats indefinitely every M-th revolution harmonics.

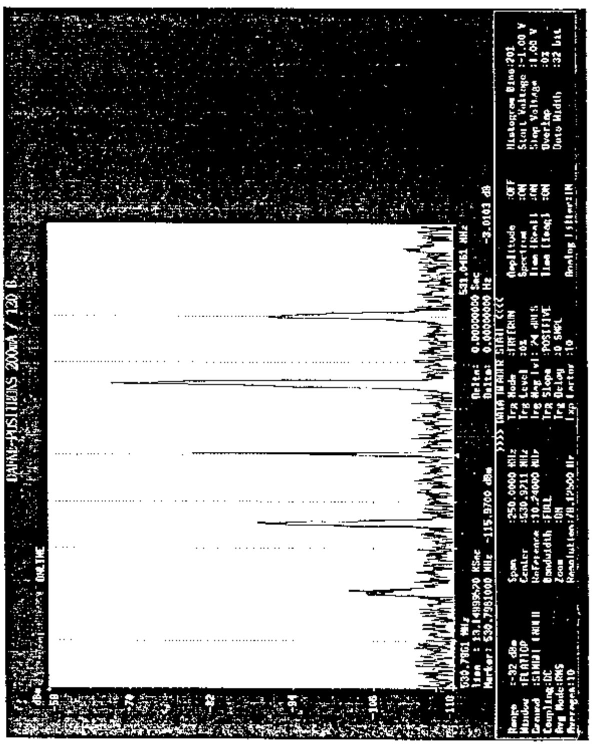
Among the sinusoids fitting the detected oscillation, those with $(p \geq 0)$ present the same slope of the actual oscillation, while those with $(p < 0)$ have an opposing slope.

We can understand that
if the real part of the impedance of a narrow band resonator has a large value at a CB mode frequency line,
the bunch motion can be excited or damped,
according to the relative slope of the motion and of the induced voltage in the resonator.



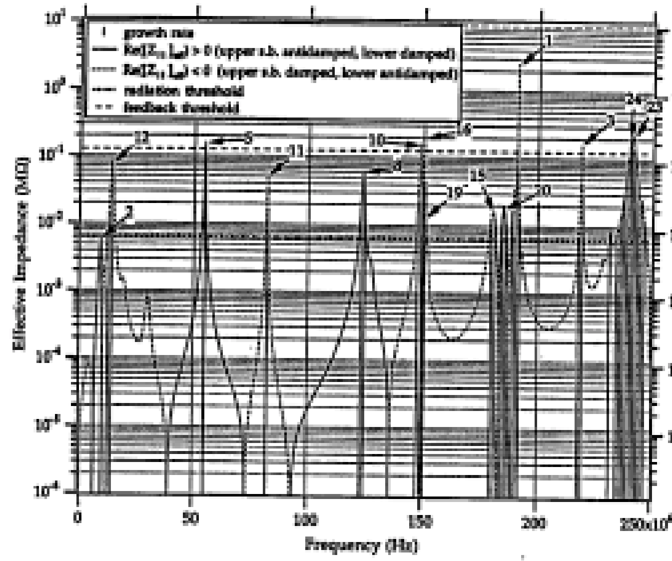


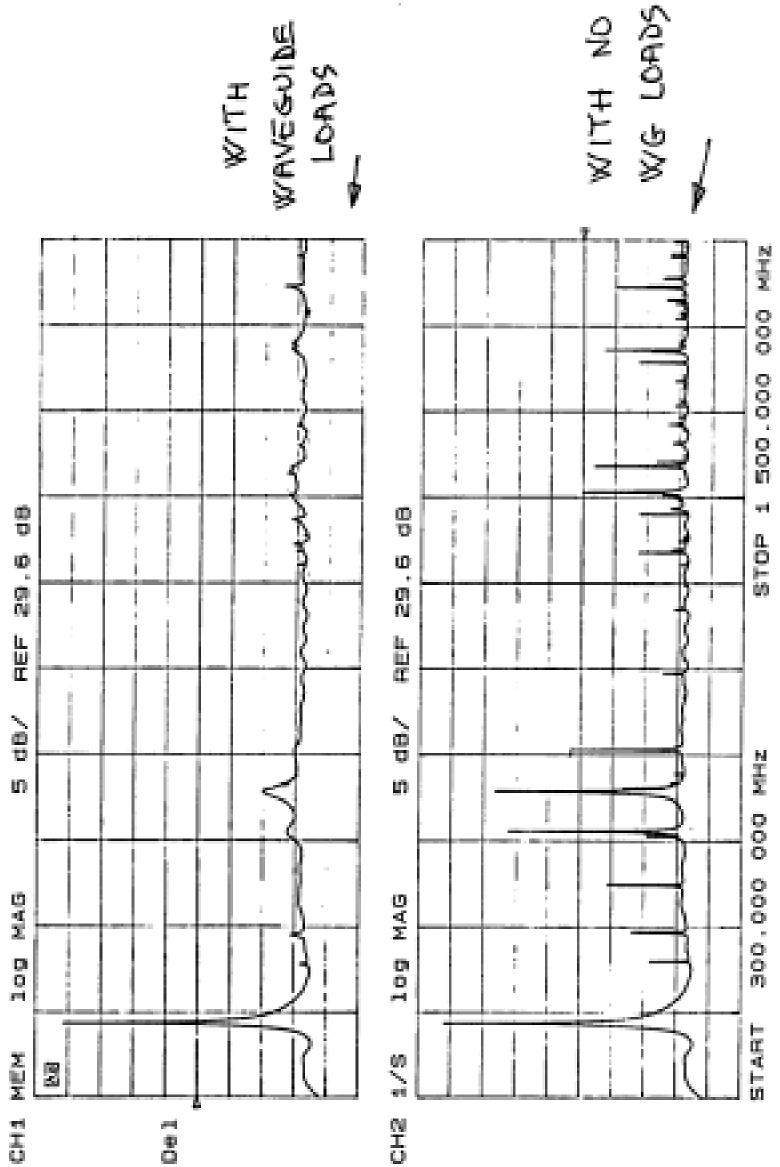




GROWTH RATES

Since all coupled modes appear in a frequency interval $Mf_0/2$, the growth rates α_n and the sum of offending impedances can be represented in an *aliased* way in the interval 0 -$Mf_0/2$.





IMPEDANCE SPECTRUM OF DAΦNE CAVITY

NECESSARY FEEDBACK GAIN

HOM GROWTH RATE :

$$\alpha_{\mathrm{HOM}n} = \frac{1}{\tau_{\mathrm{HOM}n}} = \frac{1}{2} I_0 \frac{\alpha_c}{E_0 Q_s} \sum_p f_{p,n} \Re\left(Z_{\mathrm{HOM},p,n}\right)$$

FEEDBACK DAMPING RATE :

$$\alpha_{FB} = \frac{1}{\tau_{FB}} = \frac{1}{2} f_{RF} * \frac{\alpha_c}{E_0 Q_s} g \sin\vartheta \,;\, g\left[\frac{\mathrm{eV}}{\mathrm{rad}}\right]$$

FEEDBACK GAIN

$$g > I_0 \sum_p \frac{f_{p,n}}{f_{RF}} \Re\left(Z_{\mathrm{HOM},p,n}\right)$$

TRANSVERSE MOTION
(M Bunches)

- Coupled bunch (CB) mode frequencies:

$$\omega_{p,n} = (pM+n+Q_\beta+mQ_s)\omega_0$$

Q_β = betatron tune
$-\infty < p < \infty$
n=0,1,2,...M-1 (CB mode number)
m=0, ±1, ±2,...., (head-tail mode number)

- Note that a coherent transverse mode m=0 exists, corresponding to a dipolar transverse oscillation of the center of mass with stationary longitudinal distribution in the phase plane; on the other hand, there are no longitudinal modes at m=0.

- Transverse multibunch motion can be driven by transverse HOMs in the cavities, but in addition, modes at low frequency can be excited by the RESISTIVE WALL IMPEDANCE, which is large at low frequency. The bigger the size of the storage ring, the faster the growth rate

PICK-UP's and KICKERS

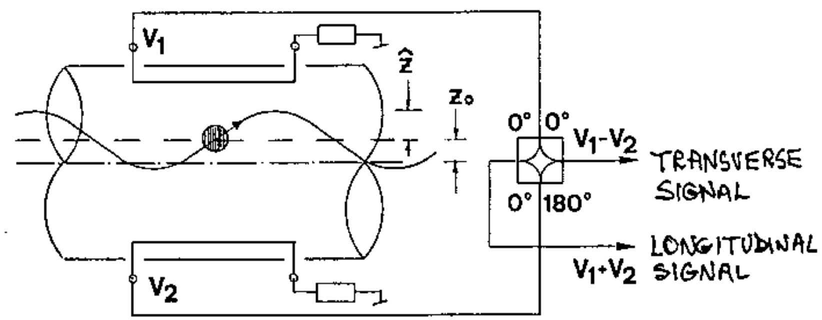
References:

- R. Littauer: "Beam Instrumentation", In "Physics of High Energy Particle Accelerators". Editor: M. Month - AIP Conference Proceedings No. 105, pp. 869-953 (1983).
- J. Borer, R. Jung: "Diagnostics", Cern Accelerator School - Antiprotons for Colliding Beam Facilities. Proceedings. Editors: P. Bryant, S. Newman - CERN 84-15, p.385 (1984).
- J.L. Pellegrin: "Review of Accelerator Instrumentation", Proceedings of the XI-th International Conference on High Energy Accelerators, CERN, Geneva, July 1980, p.459 (1980), also Slac Note SLAC-PUB-2522 (1980).
- D. A. Goldberg and G. R. Lambertson : "Dynamic Devices: A Primer on Pickups and kickers", LBL-31664 (1991).

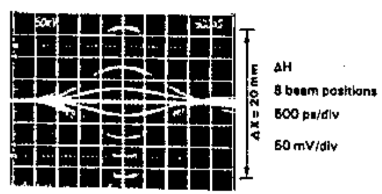
Beam Pick-up's

The pick-up transfer characteristics include the effects of the beam distance from it. By a suitable combination of pick-up's signals, it is possible to extract information about the longitudinal beam profile or its transverse position.

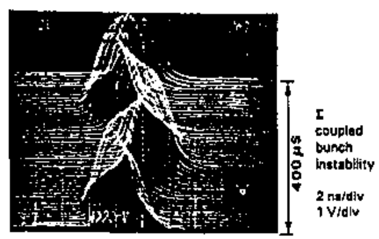
For example, by adding the signal from diametrical pickup's, one can remove the dependence on the transverse position and retain the intensity information. On the other hand, one can measure the linear dipole density (Beam Position Monitor), by subtracting the signals from two opposing pick-up's.



SIGNALS FROM A WALL-CURRENT MONITOR
(G.C. SCHNEIDER : PAC 89, p-664)



HORIZONTAL



LONGITUDINAL

PICK-UP's and KICKERS

References:

- R. Littauer: "Beam Instrumentation", In "Physics of High Energy Particle Accelerators". Editor: M. Month - AIP Conference Proceedings No. 105, pp. 869-953 (1983).
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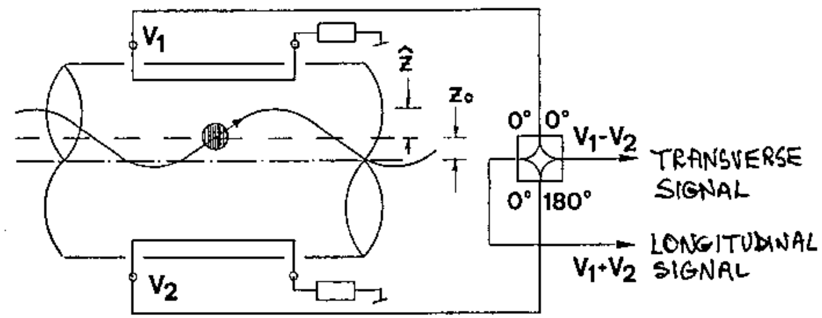
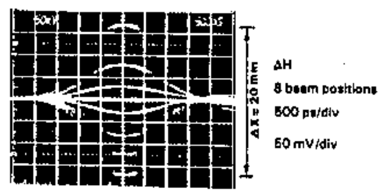
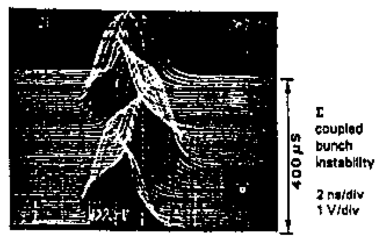


SIGNALS FROM A WALL-CURRENT MONITOR
(G.C. SCHNEIDER : PAC 89, p-664)

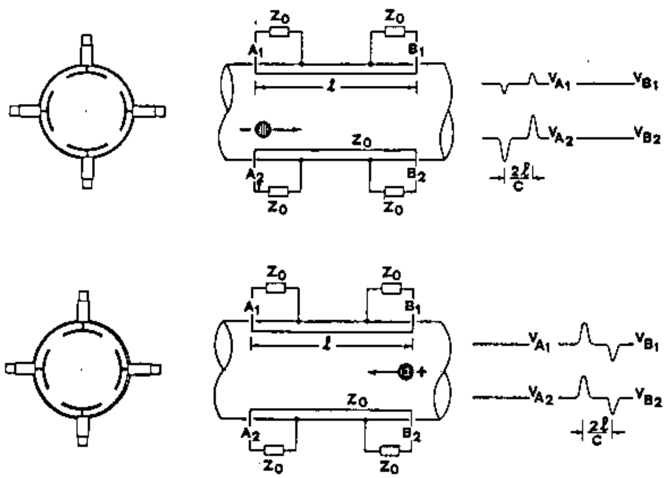


HORIZONTAL



LONGITUDINAL

Strip-Line Monitor



The strip-line is an electrode which forms with the vacuum pipe a transmission line of characteristic impedance Z_o. By a suitable choice of the ratio between the strip width and distance from the pipe, Z_o is made 50 Ω. The electrode is terminated at both ends via coaxial vacuum feed-through's into loads matched to Z_o.

Both the electric and the magnetic field contribute to the output signal but the beam electromagnetic field and the wave field in the transmission line interfere constructively at one port and destructively at the other yielding directional properties.

In principle we get an useful signal only at the up-stream port. The voltage at the up-stream load resistor is a doublet of pulses of opposing polarity and separated in time by an interval $\Delta t = 2l/c$.

No signal appears at the down-stream port as long as the beam velocity and the propagation velocity in the strip are equal (this means ultra-relativistic beam and a minimum or null amount of dielectric in the vicinity of the strip) and the load resistor is exactly matched to the line impedance. In practice, any mismatch introduced, for example, by the feed-through's or by mechanical imperfections, spoils the directional properties of the monitor.

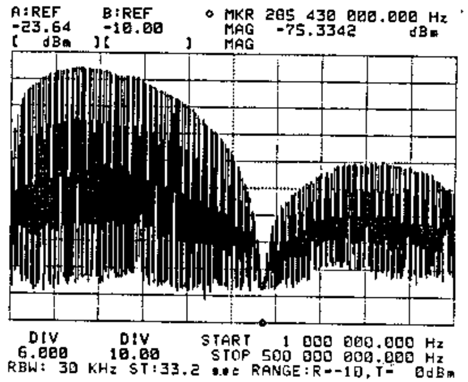
The directionality is particularly useful in colliding beams machines. One can measure only one beam's position in the presence of the other.

The time-domain voltage response of the matched strip-line is, at the up-stream port and for a centered beam

$$v(t) = \frac{Z_0}{2}\left(\frac{\alpha}{2\pi}\right)\left[i_b(t) - i_b\left(t - \frac{2l}{c}\right)\right] .$$

with α the opening angle of the strip, $(\alpha/2\pi)$ the factor of coverage and $i_b(t)$ the instantaneous beam current. The strip-line coupling impedance as a function of frequency is

$$Z_c(j\omega) \equiv \frac{V(j\omega)}{I_b(j\omega)} = Z_0\left(\frac{\alpha}{2\pi}\right)\sin\left(\frac{\omega l}{c}\right) e^{j\left(\frac{\pi}{2} - \frac{\omega l}{c}\right)} .$$



The response is maximum at frequency $f = c/4l$, or odd multiples, and zero at $f = c/2l$, or multiples.

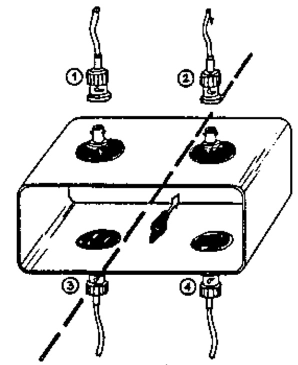
The position sensitivity of a pair of difference-connected strip-lines to a small beam displacement Δz from the center line is

$$\frac{b}{2}\frac{\Delta V}{\Sigma V} \approx \Delta z ,$$

where b is the vacuum chamber radius and ΔV, ΣV are the difference and sum voltages, resp.

Starting from the strip-line we can think of some variations (no longer directional):

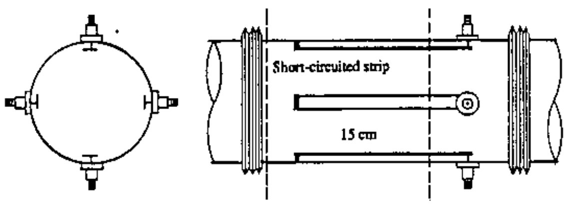
- Electrostatic monitor. A short, unterminated strip, in the form of a plate (or button), with an outside connection in the middle is mainly sensitive to the beam electric field.



$$x = k_x \left(\frac{V_2 - V_3}{V_2 + V_3} + \frac{V_4 - V_1}{V_4 + V_1} \right)$$

$$y = k_y \left(\frac{V_2 - V_3}{V_2 + V_3} - \frac{V_4 - V_1}{V_4 + V_1} \right)$$

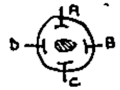
- Magnetic monitor. A strip, shorted to the vacuum chamber at the other side of the output port, forms a loop mainly sensitive to the beam magnetic field.



It is possible to obtain a tuned monitor by making the plate or loop part of an L-C resonant circuit. The sensitivity can be very high, at the expenses of a bandwidth reduction.

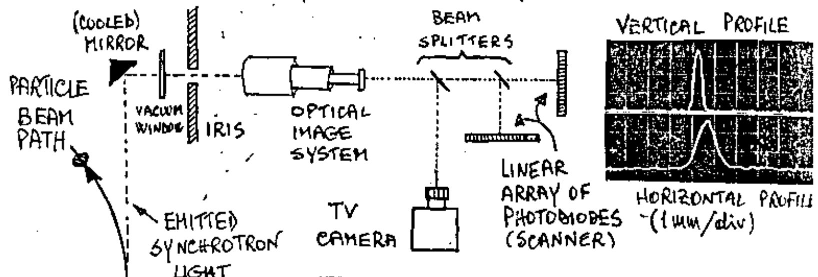
BEAM SIZE MONITORS

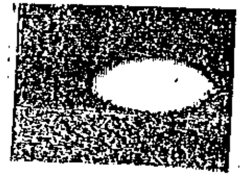
Beam size monitors can give useful information about the the incoherent (and coherent) betatron motion.
A BPM may be turned into a beam size monitor by exploiting the non-linearity of the position response:



$$\sigma_x^2 - \sigma_y^2 \propto \frac{(B+D)-(A+C)}{A+B+C+D}$$

The favourite beam size monitor, at least at electron facilities is, however, the synchrotron light monitor. Due to the high directionality of the synchrotron radiation, the intensity distribution of the emitted light reproduces fairly well the transverse charge distribution of the beam.

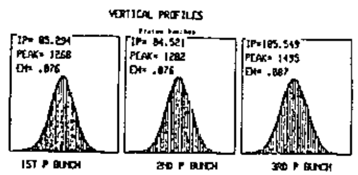


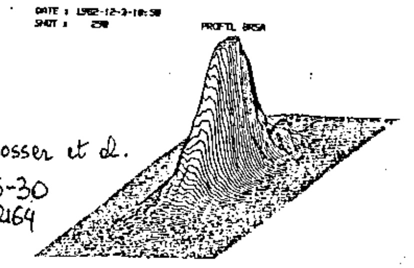


BEAM IMAGE AS OBSERVED AT TV MONITOR

Although such monitor comes naturally only with electrons its use at high-energy proton machines has been demonstrated

SPS: undulator radiation

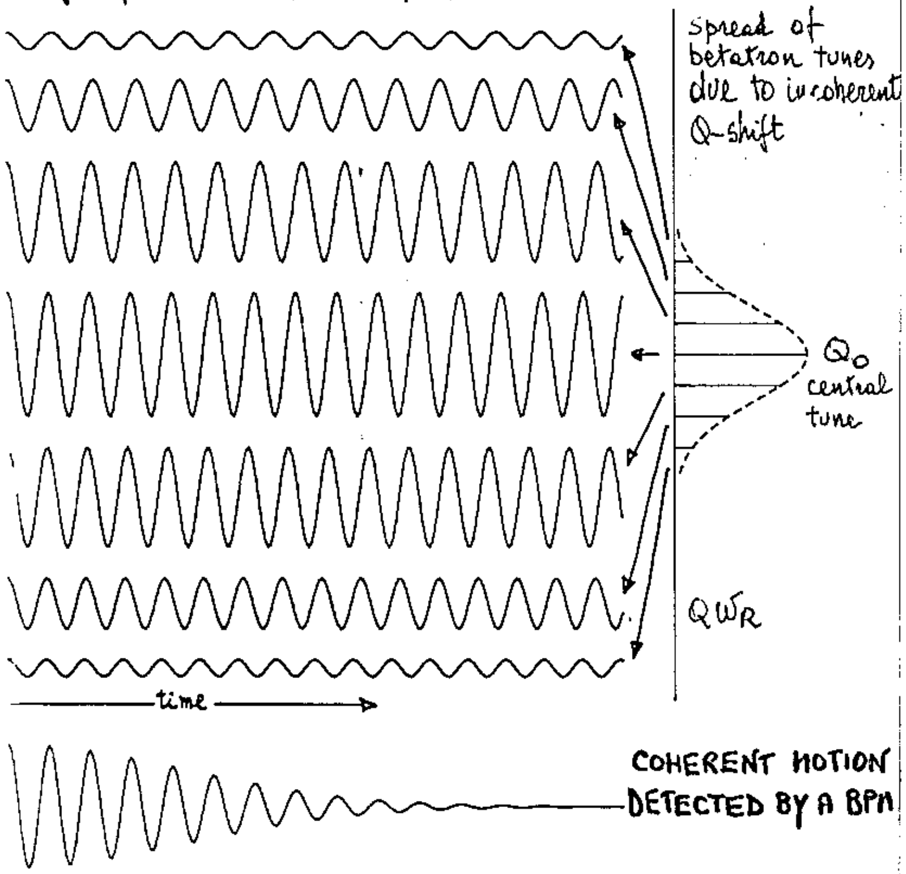




Bosser et al. NS-30 p. 2164

LINEAR SUPERPOSITION

Upon an external kick each particle starts a betatron oscillation with the same initial phase but with slightly different frequency from each other

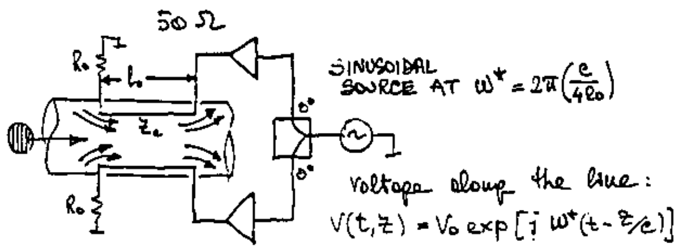


IF LOOKED AT A BEAM DIMENSION MONITOR (e.g. a synchrotron radiation monitor), THE BEAM DIMENSION TAKES MUCH LONGER TO RECOVER TO THE INITIAL VALUE (IF EVER!)

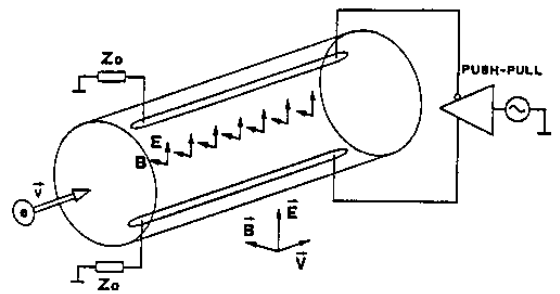
Kickers

By phase-modulating the voltage in an RF cavity it is possible to drive longitudinal dipole oscillations of a bunch. By amplitude-modulation of the RF voltage, longitudinal quadrupole oscillations can be excited, provided that the cavity bandwidth extends over the mode frequency.

We can turn a matched strip-line into a longitudinal kicker by applying in-phase deflecting voltages at the down-stream ports.

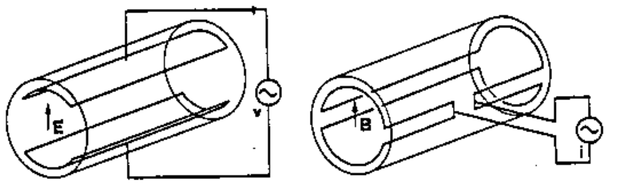


Strip-lines can also be used as transverse kicker. Two voltages of opposing polarity are applied down-stream the beam.



In the case of colliding beams, it is possible to excite selectively one beam without effect on the other.

In the same way as BPM's are sensitive to electric or magnetic field, we can deflect a beam electrically by open plates driven by a voltage generator, or magnetically, by coils driven by a current generator.



The capacitor formed by the plates and the inductor formed by the coils can be part of an L-C resonant circuit to reduce the power requirement of the driving amplifier.

Remark: by combining several strip-lines in series with $\lambda/2$ delay lines , it is possible to increase the sensitivity/strength at the peak frequency at the expense of a reduction of the bandwidth, but leaving the source/load impedance constant.

Beam Response

A basic tune measurement system can be made with a swept spectrum analyzer and a tracking generator or with a network analyzer.

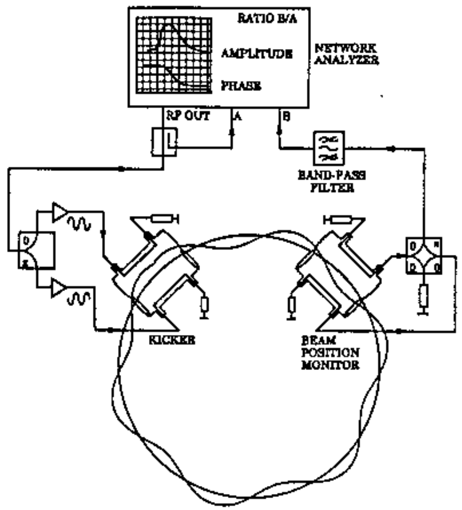
The tracking generator is a sinusoidal RF source whose output frequency exactly follows that instantaneously displayed at the spectrum analyzer. A network analyzer provides itself an RF output and measures the gain ratio and the relative phase between excitation and response altogether.

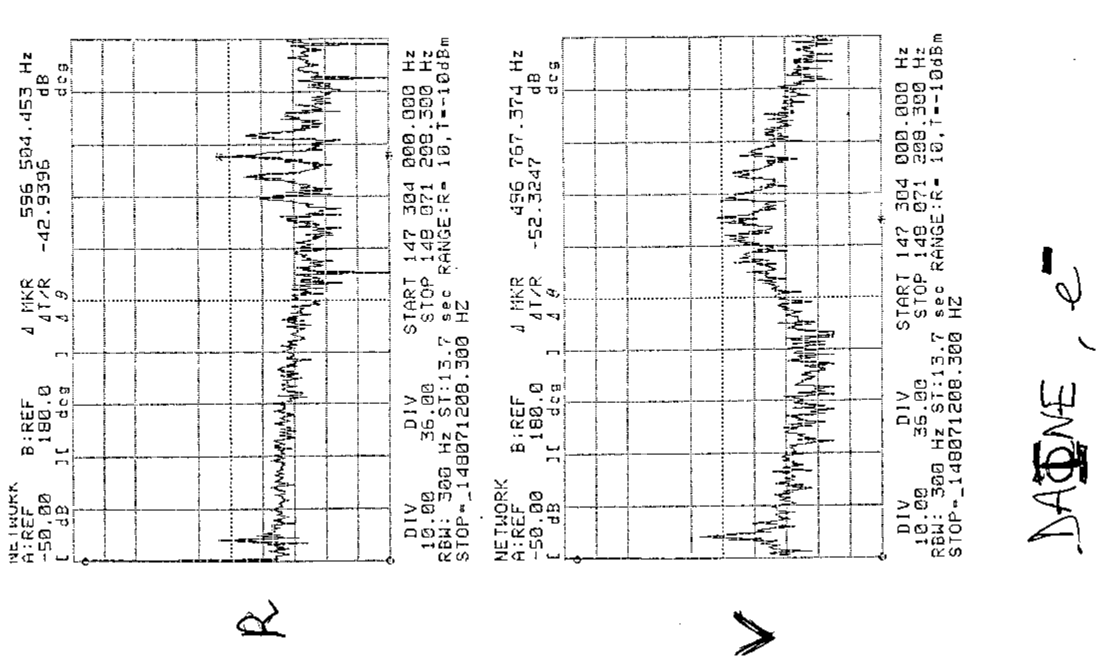
The RF output is used to drive a kicker and the signal from a beam monitor is looked at to measure the beam response (complex, with the network analyzer).

The kicker and the detector can be part of a feedback system, where available.

The tradeoff between the frequency resolution Δf and the observation time Δt is imposed by the indetermination relation

$$\Delta f \geq 1/\Delta t$$




NETWORK
A:REF -50.00 [dB
B:REF 180.0][deg
Δ MKR ΔT/R Δ θ
596 504.453 Hz
-42.9396 dB
deg
DIV 10.00
DIV 36.00
START 147 304 000.000 Hz
STOP 148 071 208.300 Hz
RBW: 300 Hz ST:13.7 sec RANGE:R= 10,T=-10dBm
STOP=_148071208.300 HZ
R
NETWORK
A:REF -50.00 [dB
B:REF 180.0][deg
Δ MKR ΔT/R Δ θ
496 767.374 Hz
-52.3247 dB
deg
DIV 10.00
DIV 36.00
START 147 304 000.000 Hz
STOP 148 071 208.300 Hz
RBW: 300 Hz ST:13.7 sec RANGE:R= 10,T=-10dBm
STOP=_148071208.300 HZ
V
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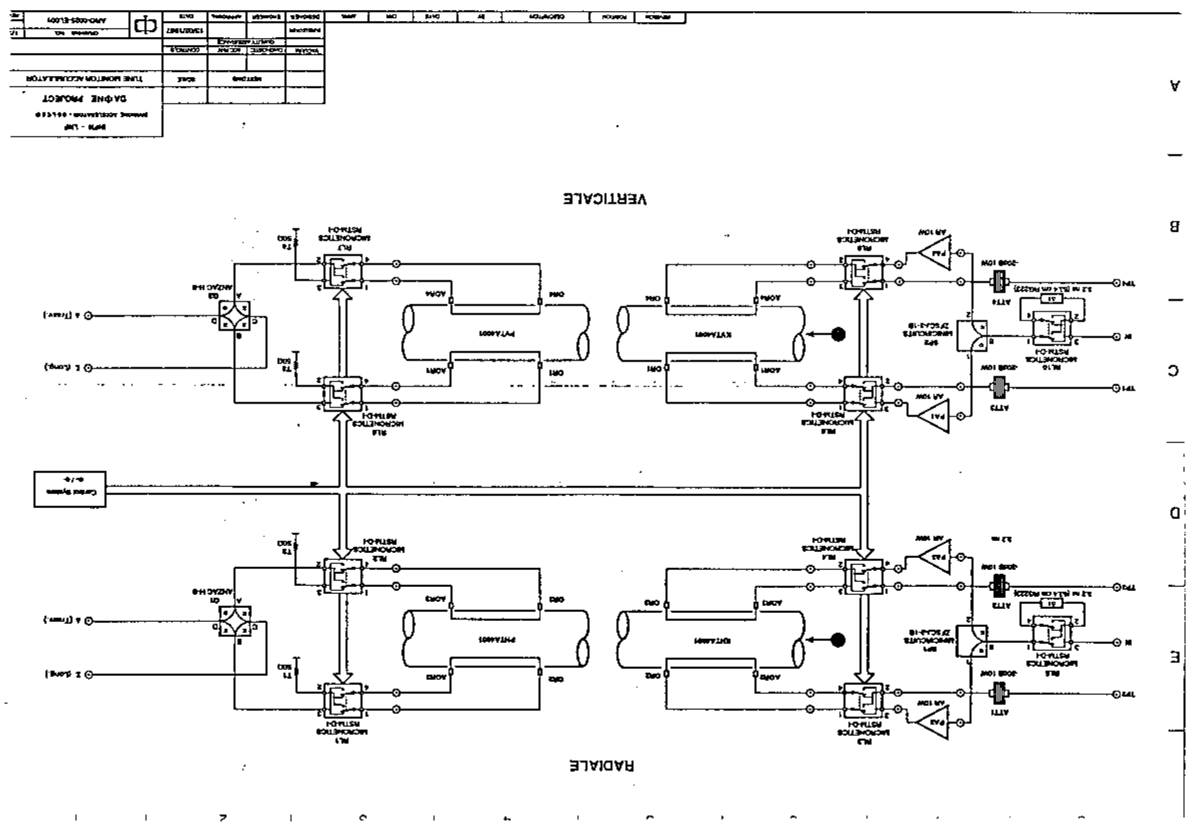
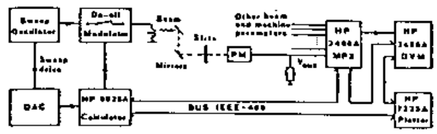

DAΦNE PROJECT
TUNE MONITOR ACCUMULATOR
VERTICALE
RADIALE

In some measurements the response of the transverse beam density is measured with a beam size monitor.



Biagini et al. HEACC 80, p687

DATA 14/05 ORE 10:58
FASCIO DI ELETTRONI (3 BUNCHES), Ibeam=62.6 mA
ENERGIA = 1500 MeV , tau damping = 11 msec
CORRENTE NEI Q-POLI "F" , 300 Amp.
CORRENTE NEI Q-POLI "D" , 385 Amp.
TENSIONE DI RADIO-FREQUENZA , 130 KV
AMPIEZZA TENSIONE DI ECCITAZIONE , 3.1 Vpp   Twait = 33 msec
SWEEP UP
Qx = 3.1250
MASSIMO ALLARGAMENTO , 5.7 X
SIDEBANDS DI SINCROTRONE A 3.7 KHZ

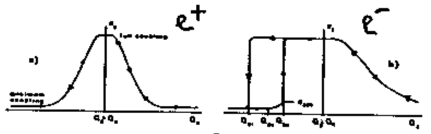
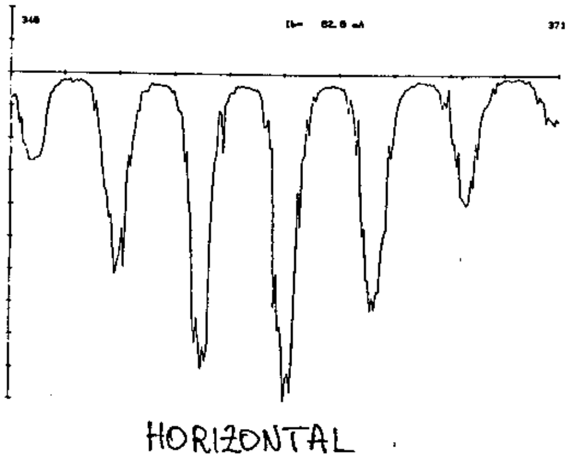
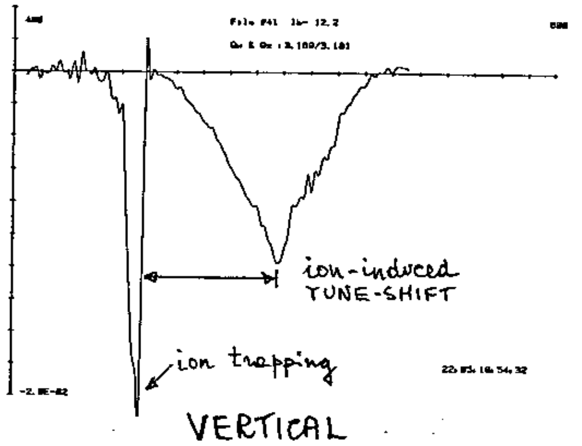
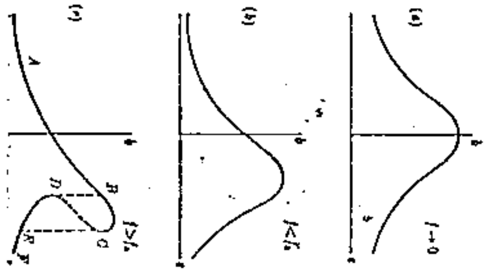


FIG. 5 - Beam vertical dimension as a function of Q_z, showing the coupling resonance: a) positrons; b) electrons.-





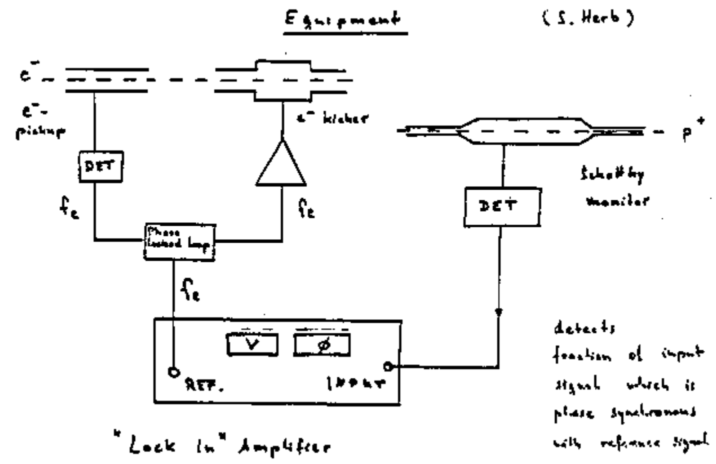


Tune Coupling

(S. Herb, F. Zimmermann - HEACC '92 - Hamburg - p. 227).

The *tune coupling* has been used very effectively to optimize the collision point of the HERA e-p colliders with separate rings. It looks very attractive for the next-generation high-luminosity factories.

The basic idea is to make a cross measurement of one beam response when the other beam is excited, as a function of an orbit bump at the IP. The beam response is at its maximum when the two beams overlap



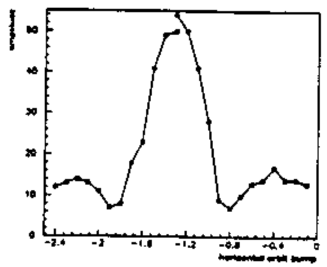


Fig. 6: Signal measured during horizontal scan; the horizontal scale is arbitrary. The solid line connects the measured points.

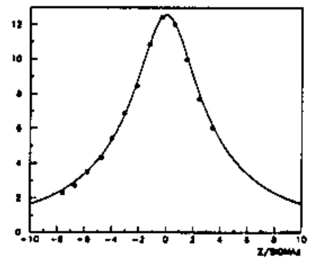
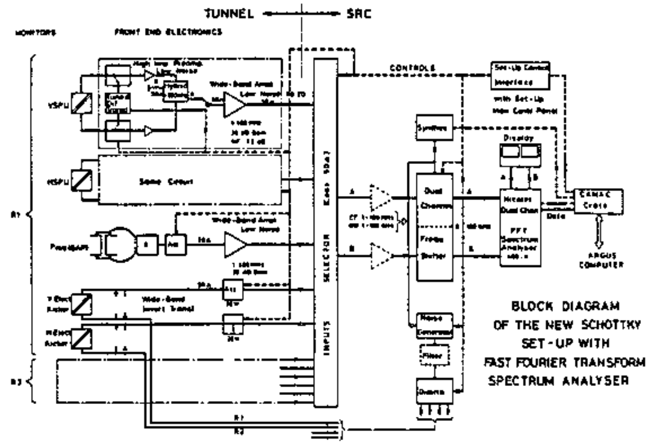
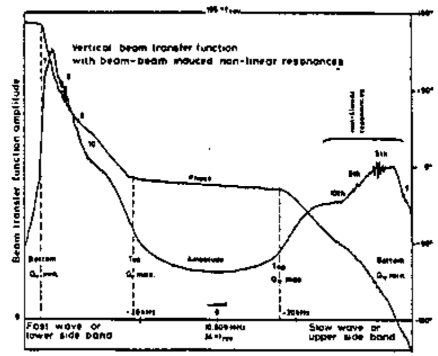


Fig. 5: Measured signal strength for vertical scan. The curve is calculated for an aspect ratio of 3.7:1

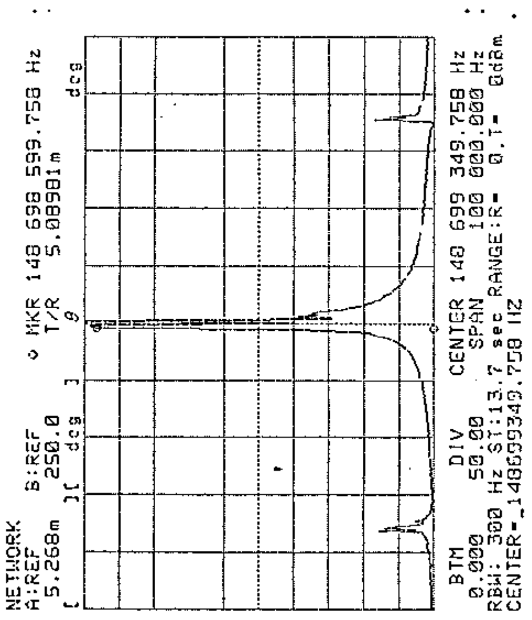
The Beam Transfer Function (BTF)

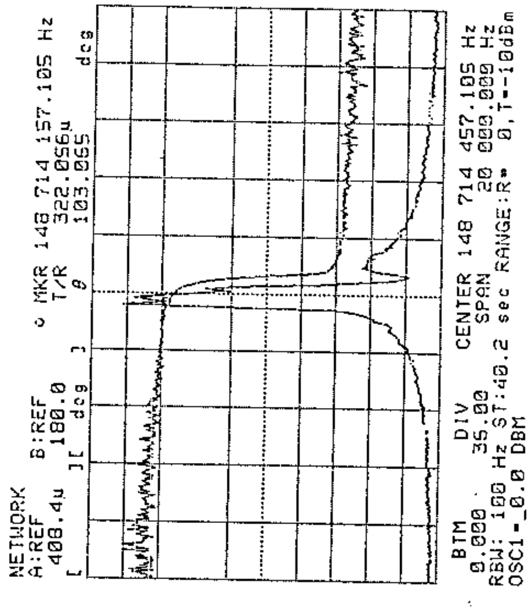
The response (amplitude and phase) of a coasting beam to an external longitudinal or transverse excitation has been used to extract information about the incoherent tune distribution, and about the forces generated by the beam interaction with the parasitic impedance of the machine and with feedback systems (J. Borer et al. : CERN - ISR - RF/80-30)





ACCUMULATOR

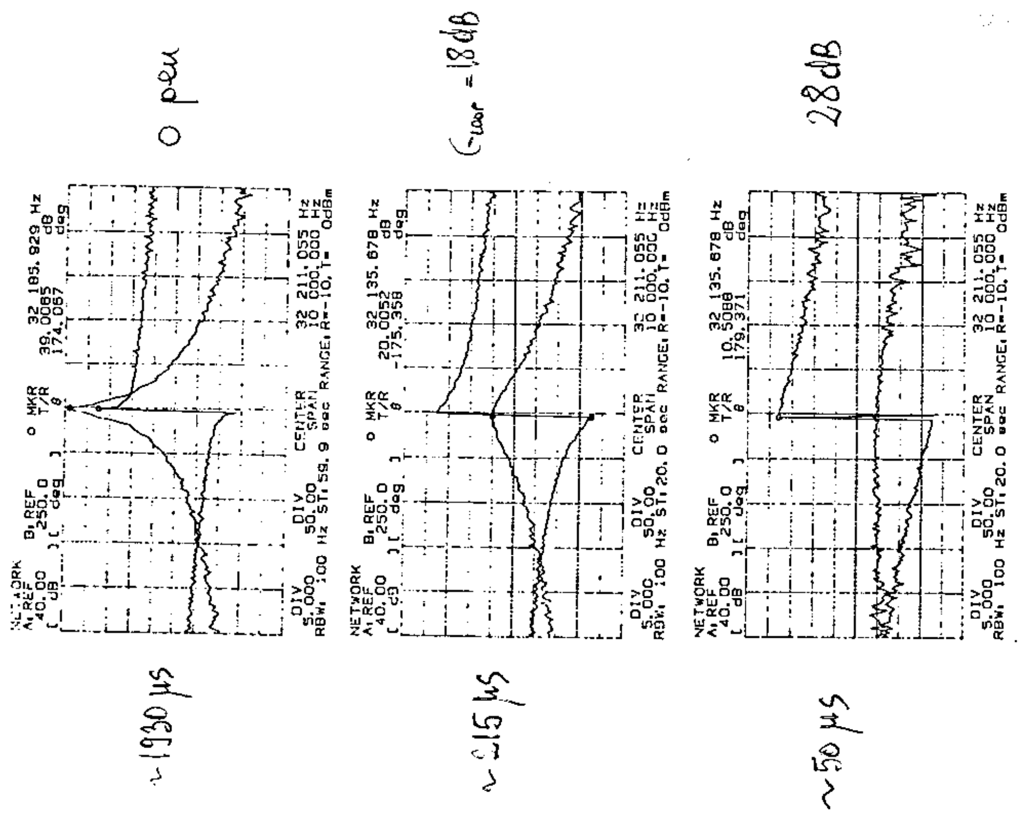


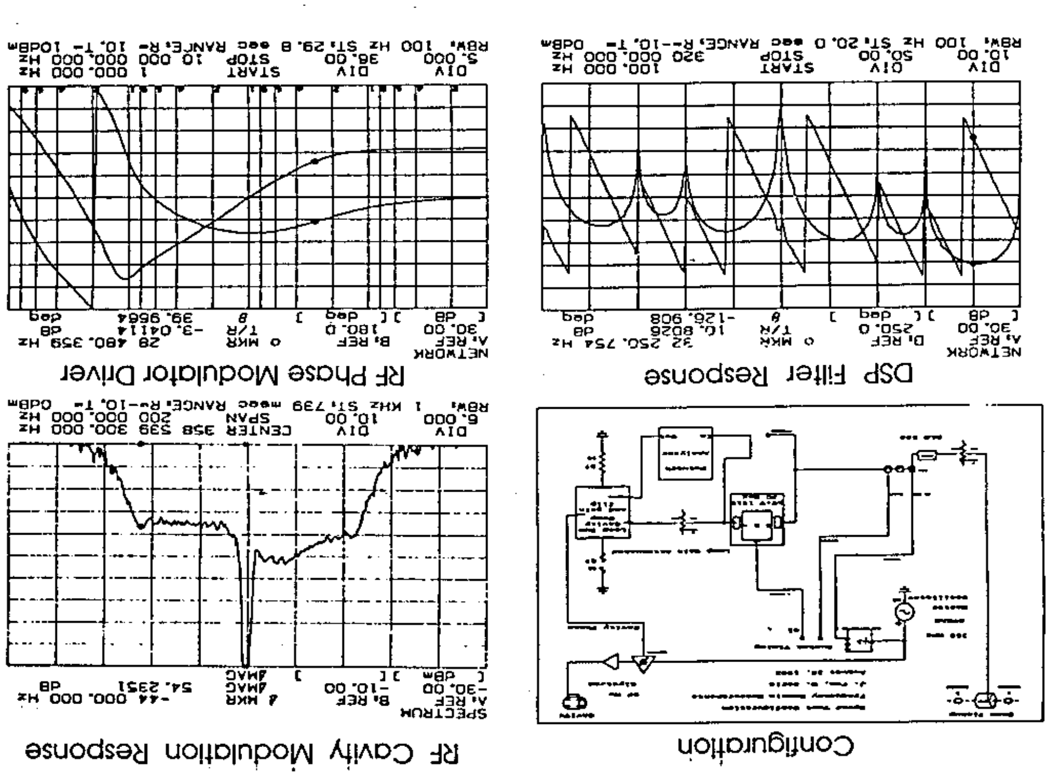


FAST

SPEAR

Beam Transfer Function

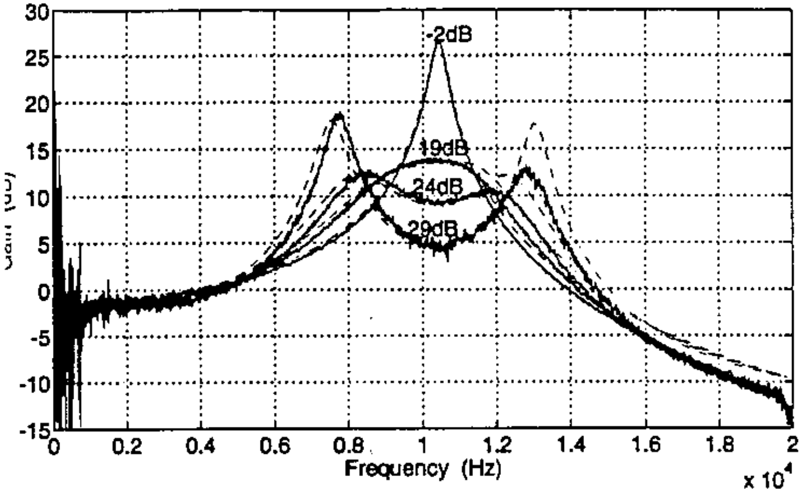




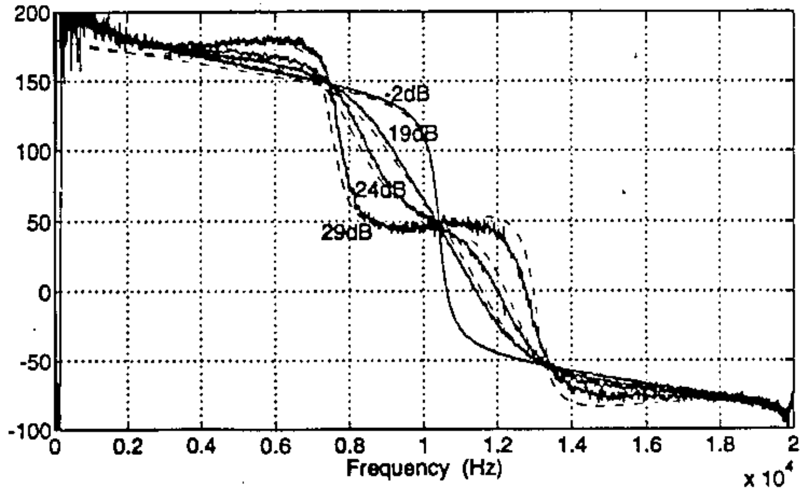
Frequency Domain Measurements (SPEAR)

ALS 4/93

Measured vs Theoretical ALS Closed Loop Frequency Responses



Measured vs Theoretical ALS Closed Loop Phase Responses



Digital Analyzers

In the measurement with a conventional swept spectrum or network analyzer, since a single frequency is analyzed at a time and because of the indetermination relation mentioned before, a long observation time is involved.

The problem can be overcome by the use of a *dynamic signal analyzer, or digital spectrum analyzer*, which is based on high-speed digital Fourier analysis (Fast Fourier Transform-FFT) executed by an embedded processor.

N voltage samples over a period T are digitized and transformed into $N/2$ complex Fourier coefficients, spanning a frequency range from DC to $N/2T$, with a frequency resolution $\Delta f = 1/T$.

The whole spectrum is available almost instantly, thus the total measurement time is reduced by a nominal factor $2/N$ with respect to a conventional swept analyzer with the same frequency resolution.

The number of frequency points is typically ~ 400, with a real-time bandwidth (no dead-time or data loss between successive spectra computations) and a dynamic range nowadays extending up to ~ 100 KHz and up to ~90 dB (good, but worse than swept analyzers) respectively.

Some analyzers provide two independent channels for spectrum analysis, a pseudo-random noise generator and capability for complex transfer function calculations.

The relatively low operating frequency is no problem, as long as the band of interest is within the maximum frequency of the FFT analyzer. For example, thᴗ IF output of a conventional RF spectrum analyzer operating in the zero-span mode as a fixed frequency detector, can be mixed down to base-band and measured at narrow resolution bandwidth with the FFT analyzer.

Beam Transfer Function - Bunched Beams

Study of the longitudinal quadrupole mode response of an electron beam (R, Boni et al: INFN-LNF-DM note RM-23) .

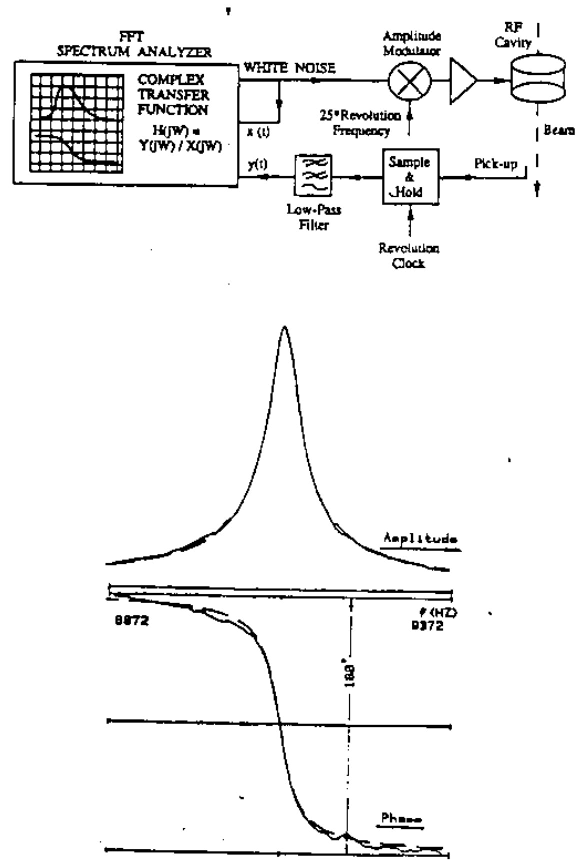
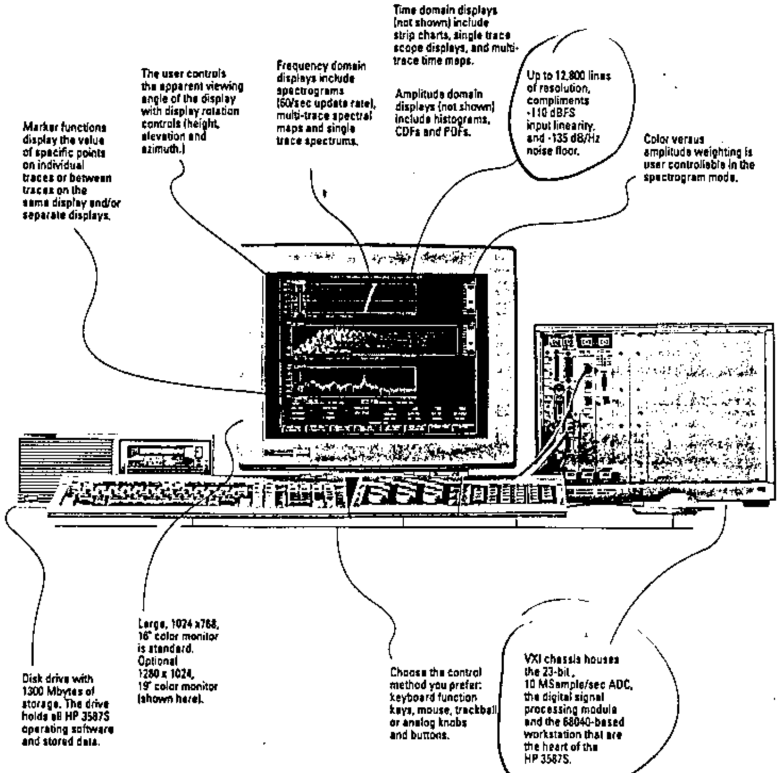


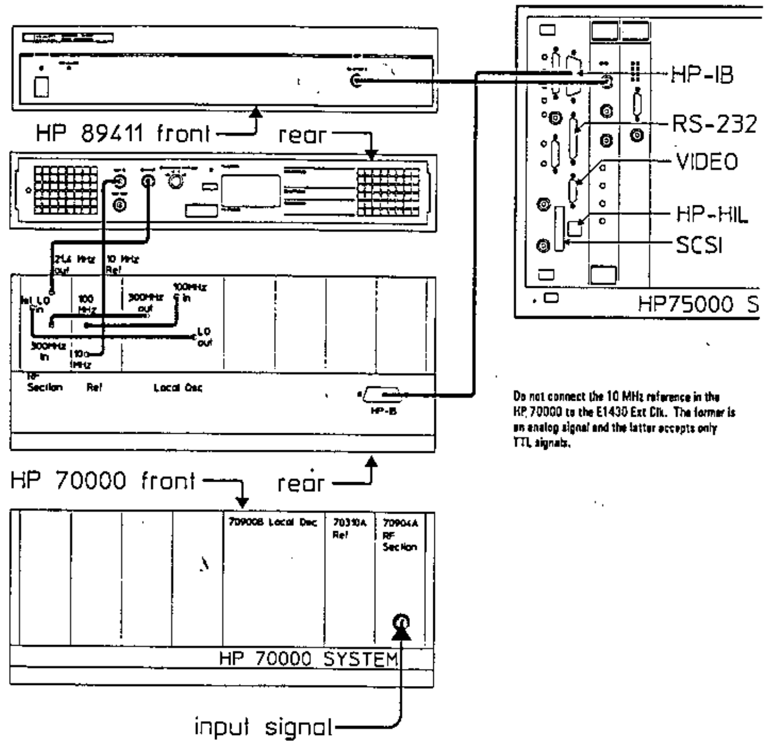
FIG. 12 - a) Schematic layout of a BTF measurement system with an FFT analyzer - b) Longitudinal BTF measurement (from ref. [23]).

HP 3587S
Measurement power and flexibility



NEW GENERATION : DSP ANALYZER
23 bit
10 Msample/sec
110 dB dyn range

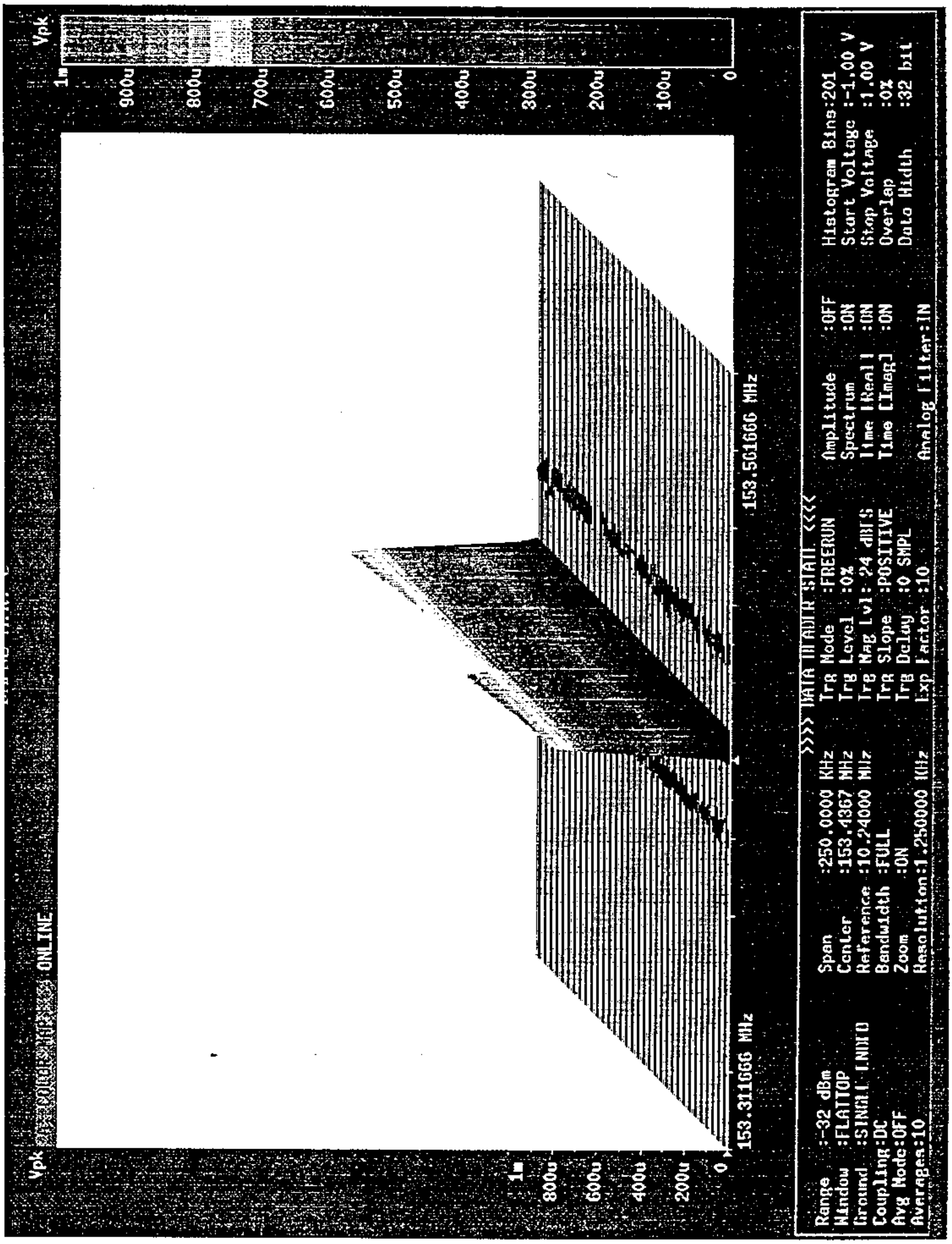
Using a Downconverter

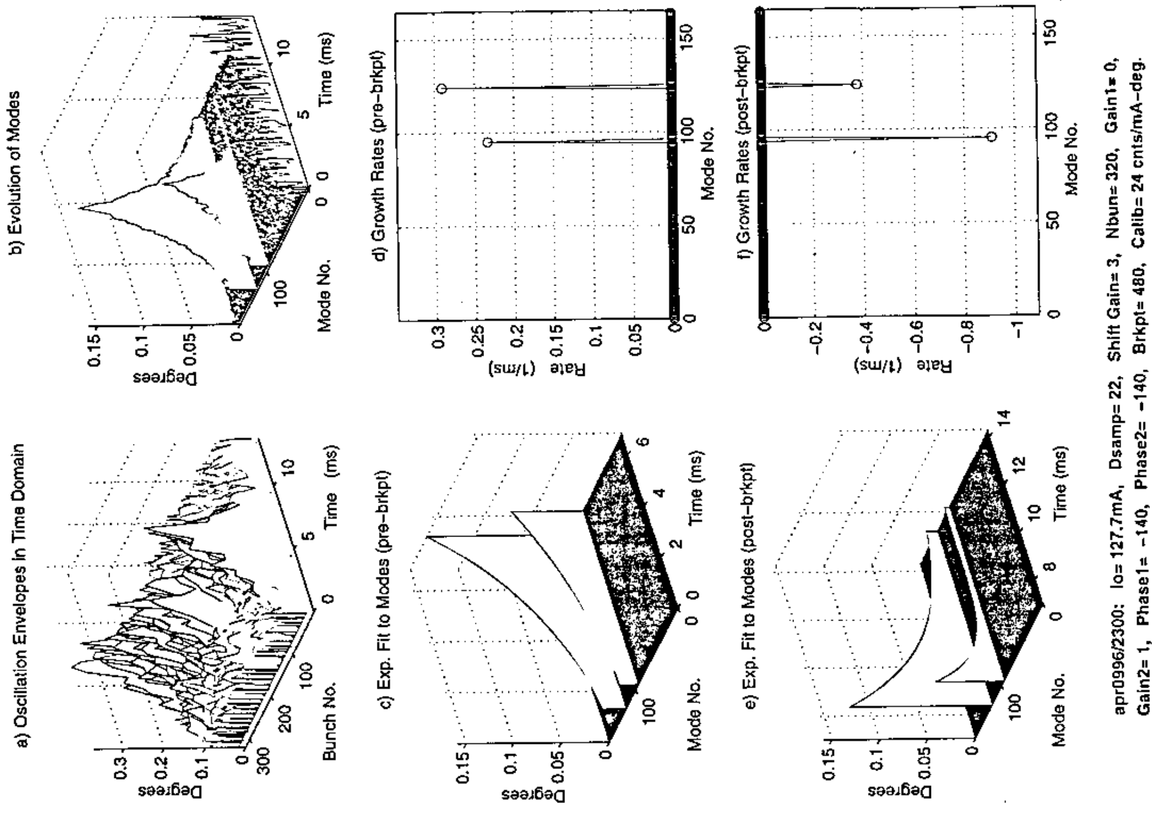


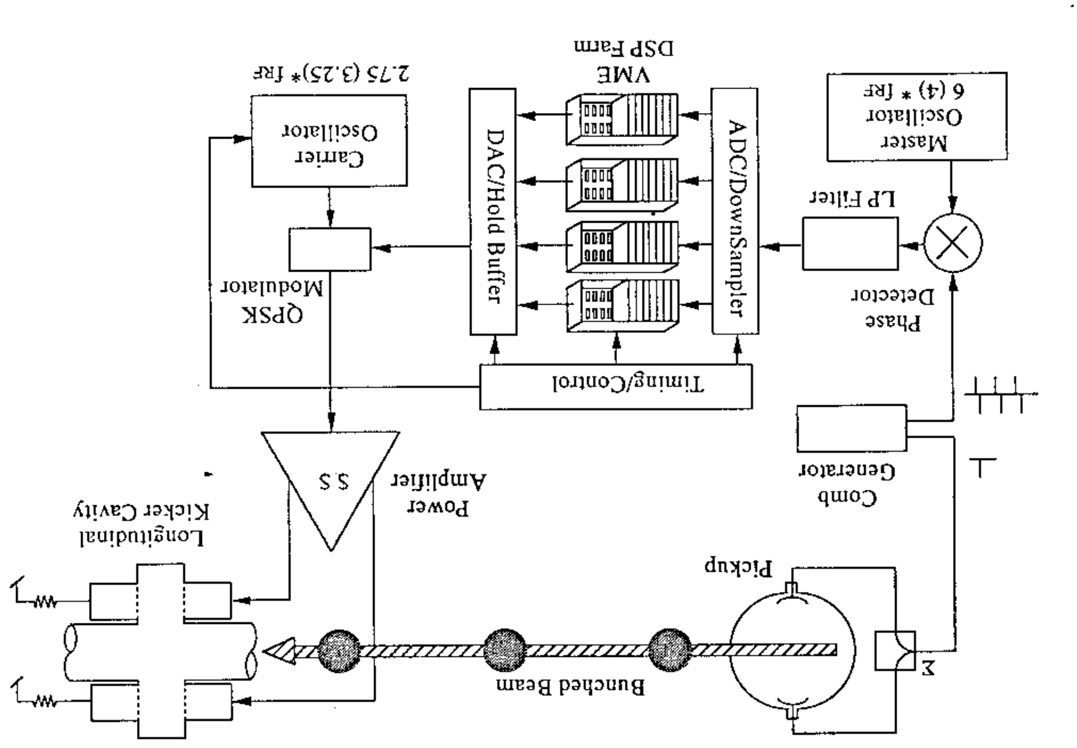
Do not connect the 10 MHz reference in the HP 70000 to the E1430 Ext Clk. The former is an analog signal and the latter accepts only TTL signals.

Configuring the HP 70000 and HP 89411 downconverters

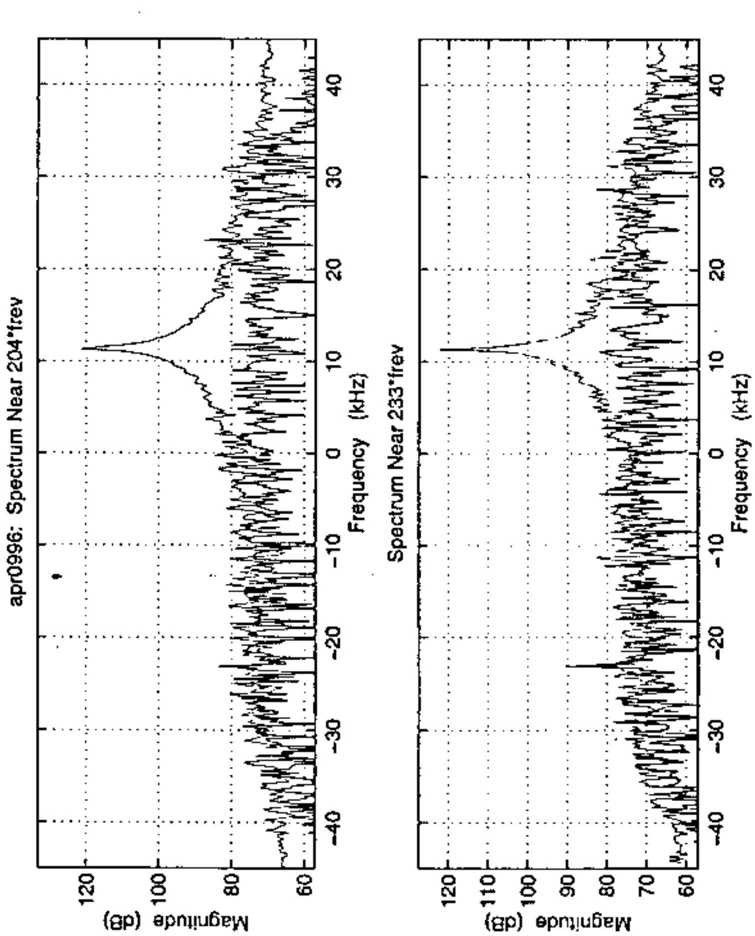
↑ RECEIVER


Vpk
ONLINE
Vpk
1m
900u
800u
700u
600u
500u
400u
300u
200u
100u
0
1m
800u
600u
400u
200u
0
153.311666 MHz
153.561666 MHz
>>>> DATA HEADER START <<<<
Range :-32 dBm
Window :FLATTOP
Ground :SINGLE ENDED
Coupling:DC
Avg Mode:OFF
Averages:10
Span :250.0000 KHz
Center :153.4367 MHz
Reference :10.24000 MHz
Bandwidth :FULL
Zoom :ON
Resolution:1.250000 KHz
Trg Mode :FREERUN
Trg Level :0%
Trg Mag Lvl: 24 dBFS
Trg Slope :POSITIVE
Trg Delay :0 SMPL
Exp Factor :10
Amplitude :OFF
Spectrum :ON
Time [Real] :ON
Time [Imag] :ON
Analog Filter:IN
Histogram Bins:201
Start Voltage :-1.00 V
Stop Voltage :1.00 V
Overlap :0%
Data Width :32 bit





LONGITUDINAL FEEDBACK SYSTEM- BLOCK DIAGRAM


apr0996: Spectrum Near 204*frev
Magnitude (dB)
Frequency (kHz)
Spectrum Near 233*frev
Magnitude (dB)
Frequency (kHz)