DAΦNE TECHNICAL NOTE

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SIMULATIONS OF THE BUNCH-BY-BUNCH FEEDBACK OPERATION WITH A BROADBAND RF CAVITY AS LONGITUDINAL KICKER

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1. Introduction

Multibunch instabilities caused by narrow band impedances existing in the DAΦNE main rings are damped by a bunch-by-bunch feedback system [1]. A longitudinal kicker is necessary to give the energy correction to each bunch. The already proposed stripline kickers [2] have come out to have several HOMs with quite high shunt impedances that further excite the beam, and they contribute a relevant amount to the total broad band impedance [3].

We have therefore explored the possibility of using a waveguide loaded RF cavity as longitudinal kicker [4]. With the same broad band impedance this device gives a higher transfer impedance, while the waveguides, conceived to enlarge the fundamental mode bandwidth, give as a welcome by-product the damping of the cavity HOMs.

In this note we investigate the impact of this kind of kicker on the feedback efficiency by means of our time domain code simulating the multibunch longitudinal dynamics [5].

2. The simulation code

The main difference between the stripline kicker and the RF cavity from the point of view of the feedback efficiency is essentially due to the different band-width of the two devices. This means that the "memory" of the RF cavity can perturb the energy kicks given to the bunches and, depending on the value of the quality factor Q, may cause coupling between neighbouring bunches.

In Fig. 1a we have represented the block diagram used in the code. To simulate the finite filling time of the RF cavity we have used a parallel RLC model, as shown in Fig. 1b.

An external amplitude modulated RF generator of fixed frequency ω_e excites the cavity in order to give the proper energy correction to each bunch. After the passage of the (k-1)th bunch the amplitude of the external generator suddenly changes to the value of the kick pertaining to the kth bunch, while the value of the phase φ_k can remain constant or jump at the bunch passage depending on the operating conditions as explained later on in this paragraph.

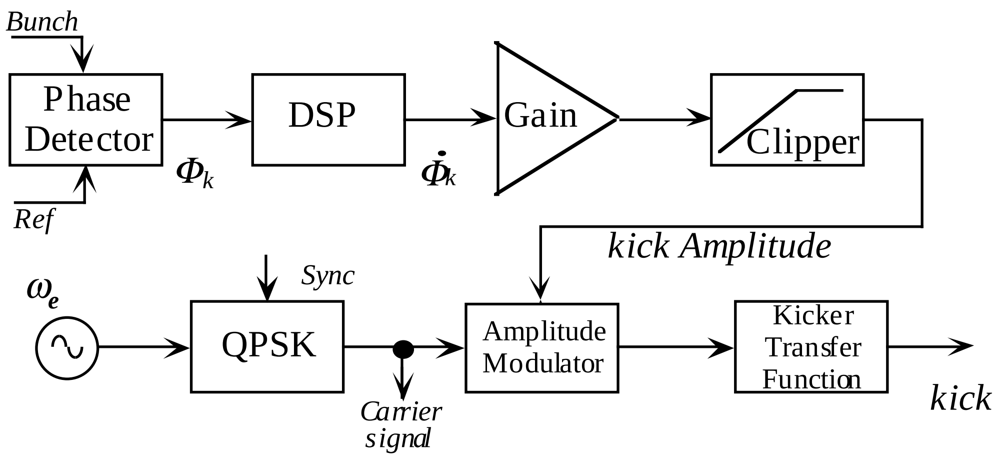

Fig. 1A: Block Diagram of the bunch-by-bunch longitudinal feedback system

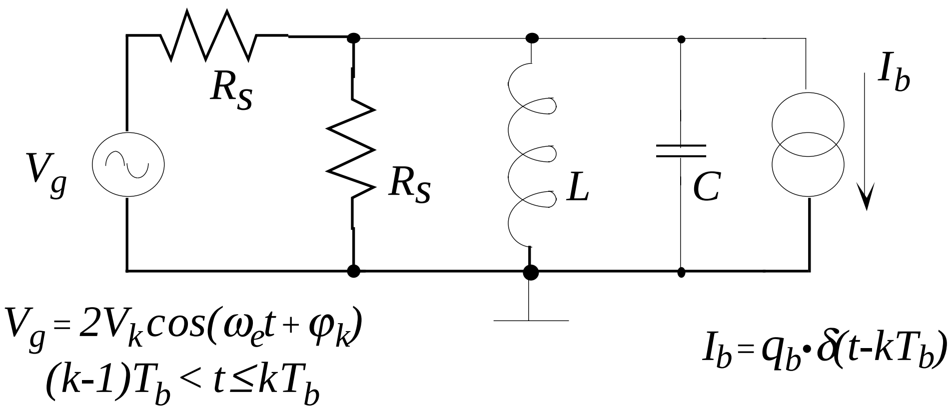

Fig. 1B: Equivalent circuit implemented in the simulation code

The total voltage seen by a particle is the sum of the voltages induced by the beam and the external generator. The first contribution can be treated just as an extra resonant impedance of the ring.

For what concerns the second contribution, if we call T_b the bunch time spacing and consider $(k-1)T_b < t \le kT_b$, we can write:

$$\begin{pmatrix} V \\ \\ \dot{V} \end{pmatrix}_t = \exp(-\alpha t) \begin{pmatrix} \cos(\beta t) + \frac{\alpha}{\beta}\sin(\beta t) & \frac{1}{\beta}\sin(\beta t) \\ -\frac{\omega_r^2}{\beta}\sin(\beta t) & \cos(\beta t) - \frac{\alpha}{\beta}\sin(\beta t) \end{pmatrix} \begin{pmatrix} V \\ \\ \dot{V} \end{pmatrix}_{(k-1)T_b} + V_k \begin{pmatrix} f(t) \\ \\ \dot{f}(t) \end{pmatrix} \tag{1}$$

with

$$f(t) = A\sin(\omega_e t + \varphi_k) - B\cos(\omega_e t + \varphi_k) + \exp(-\alpha t)\Big\{\big[B\cos(\varphi_k) - A\sin(\varphi_k)\big]\cos(\beta t) + \frac{1}{\beta}\big[(\alpha B - \omega_e A)\cos(\varphi_k) - (\alpha A + \omega_e B)\sin(\varphi_k)\big]\sin(\beta t)\Big\}$$

$$\dot{f}(t) = \omega_e B\sin(\omega_e t + \varphi_k) + \omega_e A\cos(\omega_e t + \varphi_k) + \exp(-\alpha t)\Big\{-\omega_e\big[A\cos(\varphi_k) + B\sin(\varphi_k)\big]\cdot \cos(\beta t) + \frac{1}{\beta}\Big[\big(\omega_e \alpha A - \omega_r^2 B\big)\cos(\varphi_k) + \big(\omega_r^2 A + \omega_e \alpha B\big)\sin(\varphi_k)\Big]\sin(\beta t)\Big\} \tag{2}$$

and

$$A = \frac{1}{1 + Q^2\left(\frac{\omega_e}{\omega_r} - \frac{\omega_r}{\omega_e}\right)^2} \qquad B = \frac{Q\left(\frac{\omega_e}{\omega_r} - \frac{\omega_r}{\omega_e}\right)}{1 + Q^2\left(\frac{\omega_e}{\omega_r} - \frac{\omega_r}{\omega_e}\right)^2} \qquad \omega_r = \sqrt{\alpha^2 + \beta^2} \tag{3}$$

where ω_r and α are the cavity resonant frequency and damping factor, and V_k is the amplitude of the kick pertaining to the kth bunch.

If we consider M bunches, and therefore M coupled-bunch oscillation modes, in the frequency domain we obtain a spectrum with sidebands:

$$(pM + n + \nu_s)\omega_o \tag{4}$$

where p is an integer, n is the mode oscillation number, ν_s is the synchrotron number, and ω_o is the revolution frequency. Since p can be either positive or negative, it is easy to see that a frequency range of only $M\omega_o/2$ centered around the mode $n = M/4$ contains the sidebands of all the modes. This means that in order to damp every possible coupled-bunch mode minimizing the cavity bandwidth we have to choose $\omega_r \approx M(p + 1/4)\omega_o$. For the DAΦNE case, where the harmonic number is 120, we have chosen $\omega_e = 390\omega_o = 3.25\omega_{RF}$.

The convenience of this choice can be explained also in the time domain. In fact, as shown in Fig. 2, the kick given to each bunch does not perturb the following one so that the bunch coupling due to the finite cavity filling time is weakened.

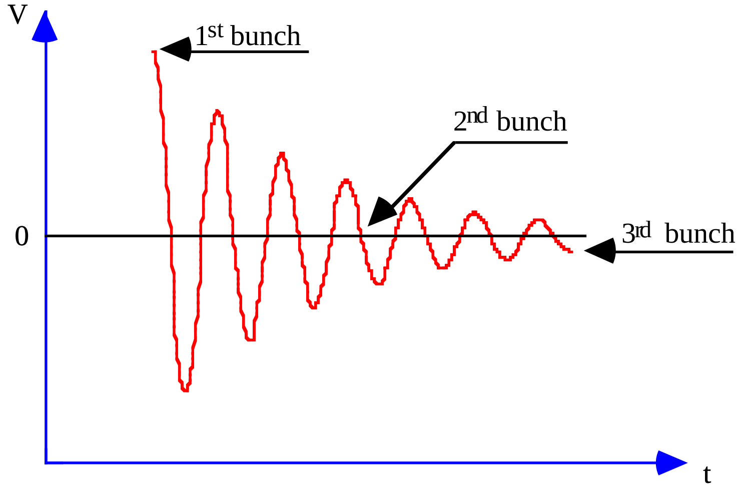

Fig. 2: Voltage given to the first bunch.

Since ω_e is not a multiple of ω_{RF} , if the phase φ_k were a constant not all the bunches could get the desired energy kick when the machine is running at full current (120 bunches in the DAΦNE case). In order to do that, it is necessary to shift the phase by $\pi/2$ every T_{RF} that is at every bunch passage. This task is accomplished by a QPSK (Quadrature Phase Shift Keyed) device introduced in the control electronics of the power amplifier feeding the kicker (Fig. 1a). Therefore just after the (k-1)th bunch has passed in the cavity, the external generator, restarting from zero, induces a voltage with an amplitude equal to the energy kick pertaining to the kth bunch. When the kth bunch enters the cavity it gets the kick and the generator starts again from zero, and so on.

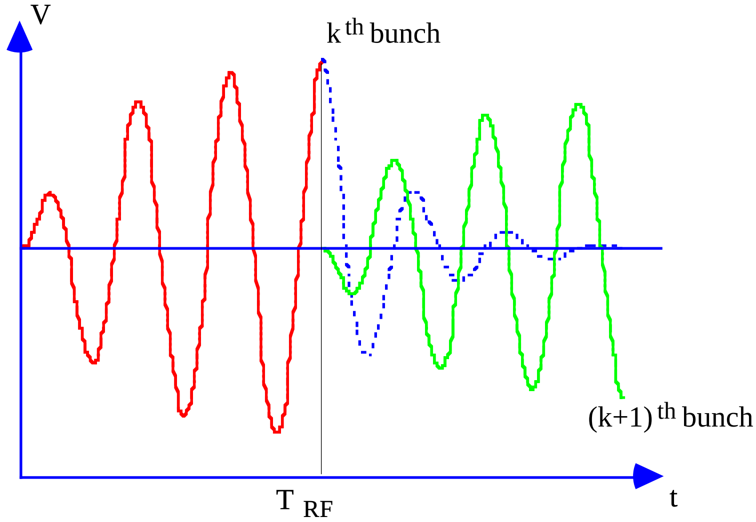

Fig. 3: Time evolution of the voltage in the RF kicker.

The plot of the kicker voltage is shown in Fig. 3. The dotted and solid lines represent respectively the 1^{st} and 2^{nd} terms of eq. (1).

In the 30 bunch operation the bunch spacing is equal to $4T_{RF}$ and therefore it is not necessary to provide the QPSK phase jump because, considering $\varphi_k =$ const = $\pi / 2$ in eq. 1, every bunch enters the cavity when the voltage is at the maximum value (ω_e is a multiple of $\omega_{RF} / 4$). In the simulation code we can include or exclude the phase jump at will.

In the following we will call "efficiency" ε the ratio between the actual kick V_{rk} given to the k^{th} bunch and the ideal value V_k corresponding to the amplitude of the driving generator. The efficiency depends on the operating conditions (number of bunches, coupled-bunch mode considered, kicker filling time, ...). Since the bunch is kicked during the transient regime of the cavity, the actual kick amplitude is lower than the ideal value corresponding to the kicker voltage at the stationary regime.

The kicker shunt impedance is defined as the stationary regime ratio between the square of the longitudinal voltage and the forward power at the kicker input $R_s = V^2 / 2P_{fw}$; then it is straightforward that the "actual" shunt impedance, i.e. the ratio between the square of the actual kick and the forward power at the kicker input, scales as ε^2.

3. Results with 30 bunches

The use of the QPSK is not convenient in the 30 bunch operation since it introduces some useless transient terms that reduce the feedback efficiency. In fact, by excluding the phase jump, the external generator can maintain the amplitude and phase constant for a $4T_{RF}$ period, corresponding to 7-8 filling times of the cavity kicker. Therefore there is enough time to let all the transients decay and every bunch gets the same kick it would receive with an ideal flat-band kicker, i.e. $\varepsilon \approx 1$. This means that the starting conditions of the cavity voltage do not influence the kick energy.

On the contrary, if we introduce $\pi / 2$ phase shift every T_{RF}, the starting conditions through the inertia of the cavity can influence the energy kick and the simulations show that, with a quality factor of 5.25 (which is the expected value for our kicker cavity), the efficiency is about 87% and the actual shunt impedance is reduced to 76% of its original value.

4. Results with 120 bunches

The QPSK phase jump is necessary in the 120 bunch operation. In this case the bunch spacing is just T_{RF} so that there is less than 2 cavity filling time between 2 bunch passages. As a consequence the kick of the k^{th} bunch depends on the initial conditions, that is on the voltage seen by the previous bunches. This explains why the efficiency of the feedback depends on the excited coupled-bunch mode. The results of the simulations show that the efficiency has an almost flat value higher than .82 for Q=5.25, while it would strongly depend on the coupled-bunch mode if the Q value were much higher (Q=20 for instance).

In Fig. 4 we report the two fitting curves obtained by exciting each single oscillation mode in our simulation code. The curves are symmetrical with respect to the mode 60.

An efficiency of 82% causes the actual shunt impedance to be reduced to the 67% of its original value. Since the shunt impedance of the DAΦNE cavity kicker, as given by e.m. simulation codes, is about 750 Ω, the available shunt impedance on the whole operating bandwidth is always higher than 500 Ω.

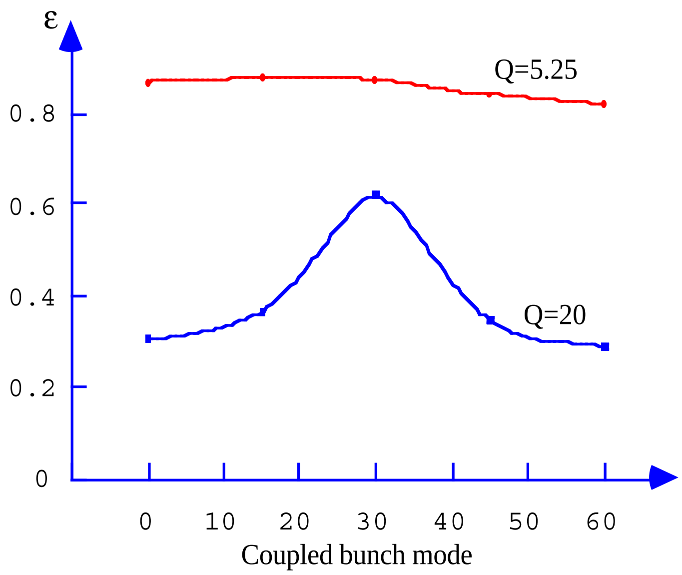

Fig. 4: RF cavity efficiency versus oscillation mode.

The efficiency curve is also reported in Fig. 5, compared to the result obtained for a cavity kicker having the same bandwidth but centered at $3\omega_{RF}$ instead of $3.25\omega_{RF}$ and driven without phase jumps. In this last case the efficiency, which is very close to 1 for the 0^{th} coupled-bunch mode, gradually decreases down to 0.75 for the mode 60. The available impedance for this last mode would be about 420 Ω.

The bunch-by-bunch longitudinal feedback system has to cure in principle any possible coupled-bunch mode with the same effectiveness; this explains why the configuration characterized by $\omega_r = 3.25\omega_{RF}$ and operated with the QPSK phase jumps is preferred.

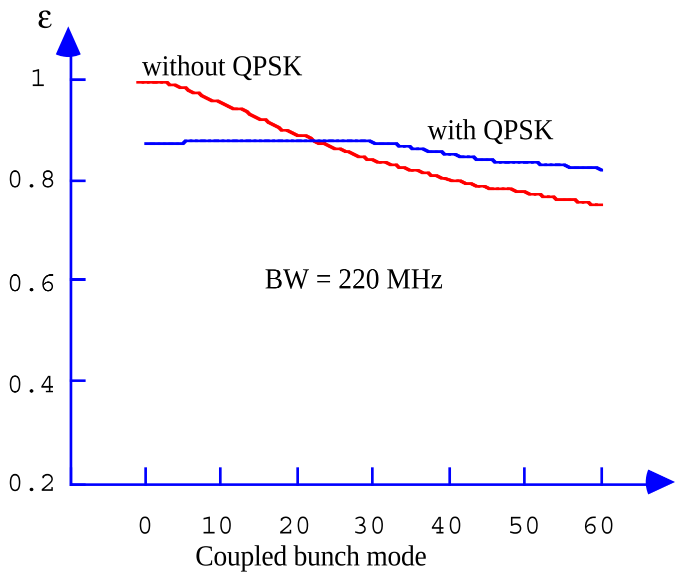

Fig. 5: RF cavity efficiency with and without QPSK.

The efficiency can also be calculated analytically as shown in the appendix. The results are reported in Fig. 6. In the plot the dots represent the results obtained with the simulation code. For very low Qs the efficiency value gets close to 1.

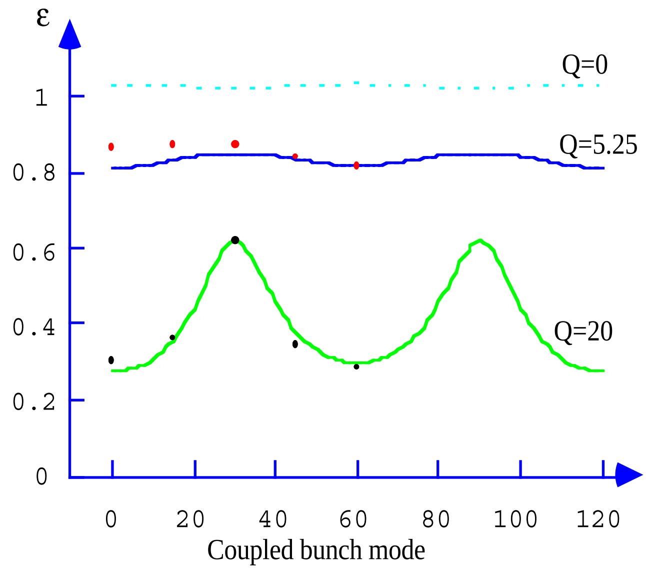

Fig. 6: Kick efficiency obtained analytically.

5. Injection error with 120 bunches

We have also simulated the longitudinal dynamics of 120 bunches injecting the last bunch with an error of 100 psec. In this case we have a superposition of all the multibunch oscillation modes. For the first turns we have also the saturation of the feedback system. The time domain responses of an ideal flat-band kicker and the band limited RF cavity (Q=5.25) are shown in Fig. 7. We can see that also in this case the response difference is about 20% both in the linear and in the saturated regime.

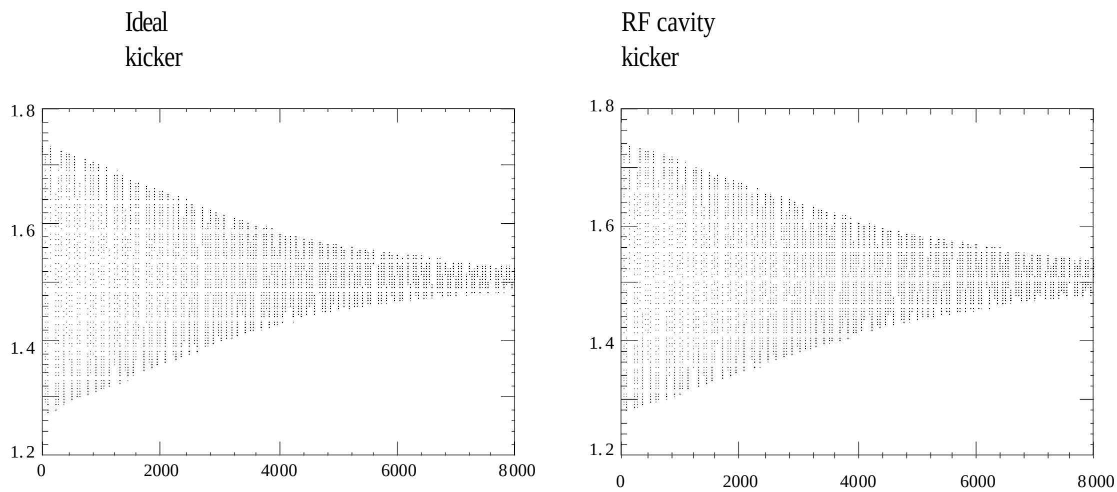

Fig. 7: Bunch oscillations at injection.

5. Conclusions

The time domain simulation code has been used for evaluating the impact of a finite bandwidth kicker on the beam longitudinal dynamics. The simulations have shown an almost flat response (Q=5.25) for all the oscillation modes. With 30 bunches the feedback system does not need to be operated with phase jumps in the driving voltage and the efficiency is practically equal to the case of an ideal flat-band kicker. The transmission efficiency is instead higher than 0.8 for 120 bunches. This value can be used to estimate the power of the amplifiers necessary to feed the cavity.

The transfer impedance of the broadband RF cavity ($\geq$ 500 Ω on the whole operating bandwidth) is higher than that of a stripline kicker[2]. Moreover, a broadband cavity does not introduce other sharp longitudinal and transverse impedances in the ring.

The results of the simulation code are well confirmed analytically by studying the system in the frequency domain.

APPENDIX

Let us call $A(t)$ the signal of the beam detected at a given position in the ring by a pickup. The corresponding energy kick should be proportional to $A(t)$ opportunely delayed ($\pi/2$ in the longitudinal phase space). If we consider M bunches separated by T_{RF} that are oscillating longitudinally, we can write for the oscillation mode n:

$$A(t) = A(qT_{RF}) = \sum_{m=1}^{M} \cos\left[\omega_s qT_{RF} + \frac{2\pi}{M}(m-1)(n-\nu_s)\right]\delta_{q,kM+m-1} \tag{A1}$$

where we have evaluated the signal every RF period and $\delta_{h,k}$ is the Kronecker delta. This signal is processed by the feedback system and, by ignoring the phase delay, we can assume that the energy kick in the ideal case is proportional to $A(qT_{RF})$.

Also in the RF cavity the external generator induces a voltage proportional to $A(qT_{RF})$. But in this case the voltage oscillates with a frequency ω_e. Due to the QPSK, the signal that enters the cavity during a period between qT_{RF} and $(q+1)T_{RF}$ can be written as:

$$S_q(t) = A(qT_{RF})\sin\left[\omega_e(t-qT_{RF})\right]\left[H(t,qT_{RF}) - H(t,(q+1)T_{RF})\right] \tag{A2}$$

where $H(t,qT_{RF})$ is the Heaviside step function. The Fourier transform of this signal is:

$$S_q(\omega) = \frac{1}{\omega_o}A(qT_{RF})\exp(-i\omega qT_{RF})F\left(\frac{\omega}{\omega_o}\right) \tag{A3}$$

with

$$F\left(\frac{\omega}{\omega_o}\right) = \frac{\omega_o^2}{\omega^2-\omega_e^2}\left\{\exp\left(-i\frac{2\pi}{M}\frac{\omega}{\omega_o}\right)\left[\frac{\omega_e}{\omega_o}\cos\left(\frac{2\pi}{M}\frac{\omega_e}{\omega_o}\right) + i\frac{\omega}{\omega_o}\sin\left(\frac{2\pi}{M}\frac{\omega_e}{\omega_o}\right)\right] - \frac{\omega_e}{\omega_o}\right\} \tag{A4}$$

The overall signal is $\quad_{q=-\infty}$. After some manipulations we can write:

$$S(x) = \frac{F(x)}{2\omega_o}\left\{\frac{\sin[\pi(n-x)]}{\sin\left[\frac{\pi}{M}(n-x)\right]}\exp\left[i\frac{M-1}{M}\pi(n-x)\right]\sum_{h=-\infty}^{\infty}\delta(h+x-\nu_s) + \frac{\sin[\pi(n+x)]}{\sin\left[\frac{\pi}{M}(n+x)\right]}\exp\left[-i\frac{M-1}{M}\pi(n+x)\right]\sum_{h'=-\infty}^{\infty}\delta(h'+x+\nu_s)\right\} \tag{A5}$$

where $x = \omega/\omega_o$ and δ is the usual delta function.

To obtain the cavity voltage we have to multiply the signal by the cavity transfer function. The cavity voltage in the time domain is obtained by the Fourier inverse transform and can be written as:

$$V(t) = \frac{1}{4\pi} \sum_{h=-\infty}^{\infty} \left\{ F(-h+\nu_s) Z\left[\omega_o(-h+\nu_s)\right] \exp\left[i \frac{M-1}{M} \pi(n+h-\nu_s) \right] \exp\left[i\omega_o(-h+\nu_s)t \right] + F(h-\nu_s) Z\left[\omega_o(h-\nu_s)\right] \exp\left[-i \frac{M-1}{M} \pi(n+h-\nu_s) \right] \exp\left[i\omega_o(h-\nu_s)t \right] \right\} \frac{\sin\left[\pi(n+h-\nu_s)\right]}{\sin\left[\frac{\pi}{M}(n+h-\nu_s)\right]} \quad \text{(A6)}$$

The voltage evaluated at the passage of a given bunch m gives the efficiency of the RF cavity. If we evaluate the kick at the time

$$t = \left(\frac{m-1}{M} + p \right) T_o \quad \text{(A7)}$$

with p integer, we obtain the curves of Fig. 6 for the different oscillation modes. For a correct response at very low Qs, due to the Fourier expansion that approximates a discontinuity with the sum of continuous functions we have to evaluate the voltage at the time:

$$t = \left(\frac{m-1}{M} + p \right) T_o - \frac{2\pi}{\omega_e} \quad \text{(A8)}$$

that is at the maximum RF cavity voltage preceding the bunch passage (or the discontinuity).

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